A TEXT-BOOK

OF

DEDUCTIVE LOGIC.

A TEXT-BOOK

OF

DEDUCTIVE LOGIC

FOR THE USE OF STUDENTS.

BY

P. K. RAY, D.Sc. (LOND. AND EDINB.),
PROFESSOR OF LOGIC AND PHILOSOPHY IN THE PRESIDENCY COLLEGE,
CALCUTTA.

London
MACMILLAN AND CO., LIMITED
NEW YORK: THE MACMILLAN COMPANY
1900

Printed and stereotyped by C. J. CLAY and SON, January, 1886.
Reprinted March 1886. *Corrected and Reprinted* 1887, 1888.
Reprinted with corrections 1891, 1892, 1894. 1900

PREFACE.

THE present work has been mainly prepared for the use of students. An attempt has been made to explain clearly and concisely the fundamental doctrines of Deductive Logic. The work consists of three Parts, with an Introduction and an Appendix. The first chapter of the Introduction treats, in the first place, of the definition and province of Logic, and then proceeds to the special subject of the book and lays down its scope and limits. The second chapter explains the fundamental principles of Deductive Logic. The three parts then treat successively of Terms, Propositions, and Deductive Reasoning. In the chapter on Immediate Inference, a full account is given of the generally accepted forms.

The method of demonstration by circles, so extensively employed in this work, for proving both immediate and mediate inferences, is not new. "The use of circles," says Ueberweg, "as an aid in the demonstration of the doctrine of Syllogism, especially in Syllogistic proper, has been referred by modern logicians (*e.g.* by Mass, J. D. Gergonne, Bachmann, and Bolazano) to Euler. But Drobish [and Hamilton] have rightly remarked that, according to

the testimony of Lambert, Joh. Chr. Lange, in his *Nucleus Logicæ Weisiannæ*, 1712, uses circles, and that Christ. Weise, Rector of the Gymnasium at Zittau (*d.* 1708), was probably the inventor[1]." Hamilton uses circles in his *Lectures* to illustrate his demonstration of valid moods by canons and rules. Ueberweg fully adopts the method of circles in his "System of Logic and History of Logical Doctrines," and proves by this method alone the various forms of immediate and mediate inference.

In this work an account has been given of the Aristotelian and the Scholastic methods of determining valid moods, so that the reader will find in it all that is usually given on this subject in manuals of Deductive Logic.

As regards the nature of deductive inference, it is held that all deductive inference is *hypothetically necessary*,—that is, that the conclusion must be true if the premisses are true.

The chapter on Probable Reasoning and Probability treats of probable propositions and inferences. A probable proposition is shown to have its origin in a *proportional* proposition. General propositions are either universal, such as "All A is B," or proportional, such as "Nine in ten A's are B." Universal propositions are treated of in ordinary Logic; proportional propositions in Probability. Where we fail to establish universal propositions, we cannot draw inferences by the canons and rules of ordinary Logic; but if we can establish *proportional* propositions, we may still draw inferences in accordance with the laws and rules of Probability.

The Appendix is partly supplementary to the text, and partly supplies additional matter to the reader.

[1] Ueberweg's *Logic*, English Translation, p. 302.

A special feature of this work is the large number of examples given at the end of almost every chapter, or important division of a chapter. Repeated practice in applying the laws and rules of Logic to concrete examples is the most important part of the study of Logic regarded as a mental training; and it is with a view to this practice that so large an amount of space has been devoted to the exercises. Most of the examples of propositions, and many of the examples of syllogisms, have been selected from well-known authors, and given exactly in the form in which they occur in their writings. Some have been taken from other works on Logic, and some from University and College Examination Papers. The rest have been especially prepared for this work.

My best thanks are due to Mr A. W. Garrett, Principal, Dacca College, for the very valuable help I have received from him in the preparation of this work. On many important points connected both with the language and the matter of the work, I have had the advantage of his help. My thanks are also due to Mr Jagad Bandhu Laha, Head Master, Dacca Normal School, and Mr Rajoni Kant Ghose, Assistant Master, Dacca Collegiate School, who have kindly revised the proofs, and assisted me with their suggestions.

DACCA COLLEGE,
September, 1883.

PREFACE TO THE SECOND EDITION.

This edition has been carefully revised; and alterations and additions have been made wherever they appeared desirable. The chapter on "The Theory of Predication and the Import of Propositions" has been, in part, re-written. The chapter on "The Various Kinds of Terms" has been subjected to a careful revision. Appendix E, "The Nature and Province of Objective Logic," as well as some foot-notes and references have been added. I ought to add that some of these alterations and additions are due to the criticism of my reviewers, to some of whom I have referred in the body of the Work.

Dacca College,
November 29, 1885.

PREFACE TO THE FOURTH EDITION.

In this fourth edition alterations have been made and new matter has been introduced, wherever they appeared desirable. The section on the *Dilemma* has been re-written and Appendix G. "Note on Obversion and Contra-position," which may be regarded as a necessary supplement to the Chapter on Immediate Inference, has been added. I have to thank many teachers for valuable suggestions and criticisms which I have received from them; and to one of them I am also indebted for the Index which appears for the first time in the present edition.

Presidency College, Calcutta,
January, 1890.

CONTENTS.

INTRODUCTION.

CHAPTER I.

The Definition, Province, and Parts of Logic.

		PAGE
§ 1.	Logic defined from the Subjective Point of View	1
2.	From the Objective Point of View	4
3.	From the Linguistic Point of View	6
4.	The third not tenable by itself	7
5.	Hamilton adopts the first	8
6.	Mill in his "Examination of Hamilton's Philosophy," adopts the first with a qualification; and in his "Logic" he adopts the phraseology of the third, but the second in reality	9
7.	Spencer adopts the second	10
8.	The View adopted in this work	10
9.	The Relation of Logic to other Sciences	11
10.	The End and Province of Logic	12
11.	The Parts of Logic	14
12.	Deductive Logic	15

CHAPTER II.

The Fundamental Principles of Deductive Logic.

§ 1.	The Principle of Identity	16
2.	The Principle of Contradiction	17
3.	The Principle of Excluded Middle	17

		PAGE
§ 4.	A Postulate of Logic	20
5.	Mill, Hamilton, and Ueberweg	20
6.	Other Principles	22

PART I.—TERMS.

CHAPTER I.

The Various Divisions of Terms.

§ 1.	Name, Concept, Conception, and Term defined. A Tabular View of various Divisions of Terms	21
2.	The first division of Terms into Single-worded and Many-worded	27
3.	The second division into Singular, General, and Collective	28
4.	The third division into Abstract and Concrete	30
5.	The fourth division into Positive, Negative, and Privative	36
6.	The fifth division into Correlative and Absolute	36
7.	The sixth division into Connotative and Non-connotative. Ambiguous Terms	36
8.	The Objective Basis of the various Divisions of Terms	41
9.	*Exercises*	42

CHAPTER II.

The Denotation and Connotation, Division and Definition, of Terms.

§ 1.	The Denotation and Connotation of a Term defined	46
2.	The Relation between the Denotation and Connotation of a Term	47
3.	The Explanation of the Relation by Diagrams	48
4.	Exercises on Denotation and Connotation	50
5.	The Mutual Relations of Terms	51
	Exercises	54
6.	The Definition and Division of Terms	54
7.	The Rules of Definition	55
	Exercises	57
8.	The Rules of Division	58
	Exercises	62

PART II.—PROPOSITIONS.

CHAPTER I.

The Definition and Divisions of Propositions.

		PAGE
§ 1.	Proposition defined. Its essential elements: the Subject, the Predicate, and the Copula. Definition of Judgment	63
2.	A Tabular View of various Divisions of Propositions	66
3.	The first division into Categorical and Conditional, according to Relation	67
4.	The second division into Affirmative and Negative, according to Quality	70
5.	The third division into Necessary, Assertory, and Problematic, according to Modality	71
6.	The fourth division into Universal and Particular, according to Quantity	73
7.	The Four Propositional Forms A, E, I, and O, according to Quality and Quantity	75
8.	The Mutual Relations of A, E, I, and O, or Opposition of Propositions	77
9.	The fifth division into Analytic or Verbal, and Synthetic or Real, according to Import	79
10.	The Five Predicables:—Genus, Species, Differentia, Proprium, and Accidens	80
11.	*Miscellaneous Exercises on Propositions*	86

CHAPTER II.

The Theory of Predication and the Import of Propositions.

§ 1.	Statement of the Question	93
2.	Dr James Martineau's View	93
3.	Hamilton's View	95
4.	Mansel's View	96
5.	Ueberweg's View	96
6.	Mill on the Import of Propositions	97
7.	Mill on Hobbes's Theory	97
8.	Mill on the Denotative or Class Theory	98

PAGE

§ 9. Mill on Hamilton's Equational View and the Doctrine of the Quantification of the Predicate 98
10. Mill's own View 101
11. A few Remarks on Mill's View 104
12. Classification of the various Views into (1) Predicative, (2) Denotative, (3) Connotative, (4) Denotative-Connotative 106

CHAPTER III.

The Meaning and Representation of A, E, I, O, by Diagrams.

§ 1. The Meaning and Representation of A 111
2. The Meaning and Representation of E 112
3. The Meaning and Representation of I 113
4. The Meaning and Representation of O 114
5. Recapitulation 115
6. *Exercises* 116

PART III.—REASONING OR INFERENCE.

CHAPTER I.

The Different Kinds of Reasoning or Inference, with Examples 118

CHAPTER II.

Of Immediate Inferences.

§ 1. Immediate Inference defined 121
Two kinds of Immediate Inference:
(1) Immediate Inference from a Term.
(2) Immediate Inference from a Proposition.
Different forms of (2) are:—
2. I. Conversion 125
3. II. Obversion, Æquipollence, or Permutation . . 129
4. III. Contraposition 132
5. IV. Subalternation 135
6. V. Opposition 136

PAGE
§ 7. VI. Modal Consequence 140
8. VII. Change of Relation 141
9. Additional Forms of Immediate Inference 146
10. *Miscellaneous Exercises* 148

CHAPTER III.

Of Syllogisms.

§ 1. Syllogism defined. Its essential characters . . 151
2. Of Categorical Syllogisms 152
3. The Method of Testing by Diagrams: the two Axioms . 153
4. The General Syllogistic Rules 155
5. The Division of Categorical Syllogisms into Figures . 164
6. The Subdivision of Categorical Syllogisms in each Figure into Moods 167
7. The Determination of the Valid Moods in the First Figure 168
8. The Determination of the Valid Moods in the Second Figure 172
9. The Determination of the Valid Moods in the Third Figure 175
10. The Determination of the Valid Moods in the Fourth Figure 176
11. *Questions and Exercises* 177

CHAPTER IV.

The Aristotelian and the Scholastic Methods of Determining Valid Moods.

§ 1. Aristotle's *Dictum de omni et nullo* 180
2. The Valid Moods in the First Figure determined by the Dictum 181
3. Aristotle's Distinction of Perfect and Imperfect Figures 181
4. Reduction of Moods in the Imperfect Figures to the Perfect 182
5. Ostensive or Direct Reduction 183
6. Indirect Reduction, or *Reductio per deductionem ad impossibile* 187
7. *Exercises* 190

CHAPTER V.

The Various Kinds of Syllogisms.

		PAGE
§ 1.	The various Kinds or Divisions of Syllogisms	192
	The Subdivisions of Pure and Mixed Syllogisms	193
2.	I.—Of Pure Syllogisms:	
	i.—Categorical	193
	ii.—Hypothetical	193
3.	II.—Of Mixed Syllogisms:	
	i.—Hypothetical-categorical	195
4.	ii.—Disjunctive-categorical	200
5.	iii.—Conjunctive-disjunctive, or the Dilemma	202
6.	*Exercises*	208
7.	Of Enthymemes	210
8.	*Exercises*	211

CHAPTER VI.

Of Trains of Syllogistic Reasoning.

§ 1.	A Train of Syllogistic Reasoning, Synthetical or Analytical	216
2.	The Synthetical and the Analytical Method in Deductive Logic	217
3.	Sorites and Epicheirema, or Abridged Trains of Syllogistic Reasoning	218
4.	Symbolical Examples of Sorites, with Analyses	222
5.	*Questions and Exercises*	224

CHAPTER VII.

Of Fallacies.

§ 1.	I.—A General Outline	226
	A Tabular View of Inferential Fallacies	226
	A Tabular View of Non-Inferential but Logical Fallacies	227
	A Tabular View of Non-Logical or Material Fallacies	228
2.	II.—Fallacies in Deductive Logic	228
	A.—Logical Fallacies.	
	1.—Inferential.	
	(1)—Fallacies of Immediate Inference	229

PAGE

§ 3. (2)—Fallacies of Syllogistic Inference . . 230

2.—Non-Inferential.

4. (1)—Semi-logical Fallacies 232

5. (2)—Fallacies or Faults of Definition and Division 234

B.—Non-Logical or Material Fallacies.

6. (1)—*Petitio Principii* . . 235

7. (2)—Falsity of Premiss . . 237

8. (3)—*Ignoratio Elenchi* . . . 239

9. (4)—The Fallacies of Many Questions and *Non-Sequitur* 241

10. *Exercises* :—Directions for testing Arguments . . 242

Examples 243

CHAPTER VIII.

The Functions and Value of the Syllogism.

I.—Mill's View of the Functions and Value of the Syllogism:

§ 1. The Syllogism as a Test of Reasoning, and as an Interpreter of General Propositions 251

2. The Syllogism as involving a *petitio principii* . . 253

II.—Criticism of Mill's View:

3. The Distinction between the Psychology of Reasoning and the Logic of Reasoning 255

4. Dr Martineau's and De Morgan's Objections to Mill's View 255

5. The *hypothetically necessary* character of all Deductive Inference 260

CHAPTER IX

Probable Reasoning and Probability.

§ 1. Syllogisms according to the Modality of the Premisses . 262

2. The Meaning of a Probable Proposition 262

3. The Rules of Immediate Inference in Probability . . 265

4. The Rules of Mediate Inference in Probability:—(1) Formal and (2) Experimental . . . 266

5. The *Formal* Rules of Mediate Inference . . 266

6. The *Experimental* Rules of Mediate Inference . . 270

7. *Exercises* 273

APPENDIX.

PAGE

A.—The Canons or Axioms of the Syllogism according to Logicians:
§ 1. Lambert's Canons for the so-called Imperfect Figures:—His vindication of their independence of, and equality with, the First Figure 275
2. Thomson's Canons 279
3. Whately's Canons 280
4. Hamilton's Canons 280
5. Martineau's Canons on the Predicative View of Propositions 282
6. Mill's Canons on the Connotative View of Propositions 283

B.—The Dilemma according to Logicians 286

C.—Note on Mixed Syllogisms (or Hypothetical Syllogisms, &c. of Logicians), regarded as Immediate Inferences . 289

D.—Note on the Reduction of Inductive Reasoning to the Syllogistic Form 297

E.—The Nature and Province of Objective Logic:
§ 1. Hamilton's View:—His distinction of Subjective Logic and Objective Logic 303
2. Mill's View:—Two phases of his conception of Logic 303
3. Spencer's View:—His distinction of Logic and the Theory of Reasoning. Logic, like Mathematics, is an Objective Science, while the Theory of Reasoning is a Subjective Science 304
NOTE. Mr Carveth Read's View, Dr Venn's Criticism of Spencer's View. 307
4. Lewes's View:—His distinction of the different meanings of the word Logic and his identification of Objective Logic and Metaphysics 308
5. Summary. 311

G.—Note on Obversion and Contraposition . . . 312
Index 315

INTRODUCTION.

CHAPTER I.

The Definition, Province, and Parts of Logic.

§ 1. Logic may be defined as the science of the regulative principles of thought, that is, the science of the axioms and laws to which thought must conform in order that it may be valid. The word *science* means coherent or systematized knowledge as distinguished from unconnected or detached knowledge. Thus Algebra is a science, or a consistent body of knowledge of number, that is, of numbers and their relations; Geometry is a science, or a system of knowledge of space, that is, of the modes of space and their properties; Physics is a science of the general properties of matter; while a register of births and deaths, or of the observations of atmospheric temperature and pressure, is not a science, but a mere collection of unconnected knowledge of individual subjects and particular facts. The word *principle* means a general truth as distinguished from a particular one; the former holds good universally in all cases, while the latter is true in a single case, or in a few cases only. A principle may be self-evident like the axioms of Geometry, or proved by observation and generalization like the law of gravitation. The word *regulative* means that the principles constitute, determine or underlie all thought, that is, no thought can, properly speaking, be called thought unless it conforms to them; or, in other words,

no thought is valid unless it is conducted in accordance with them. The word *thought* is used in, at least, three senses. In the widest sense it means any mental state or phenomenon, whether of knowing, feeling, or willing. In a narrower sense it means an act or product of knowledge, whether of perception, memory, inference, imagination, &c. As used in logic, thought means sometimes the process, and sometimes the product of comparison: in the former sense it stands for conception, or judgment, or reasoning; and in the latter sense, it is a concept, or a judgment, or a reasoning. Logic treats of these processes and products, and lays down the laws and rules to which they must conform in order that they may be valid.

A concept is the product of the comparison of two or more individual things and may be viewed subjectively or objectively. Regarded subjectively, that is, as something existing in the mind, it is an idea or notion corresponding to an attribute or collection of attributes possessed in common by a number of individual things. For example, the concept 'man' is an idea corresponding to those attributes in which all individual men agree. Suppose that those attributes are 'animality' and 'rationality,' then the concept 'man' is the idea or notion corresponding to these two attributes. Similarly, the concept 'triangle' is the idea or notion corresponding to the attribute of 'being bounded by three lines,' possessed in common by all triangles; the concept 'horse' is the idea or notion corresponding to the collection of attributes in which all horses agree; the concept 'animal' is the idea or notion corresponding to the attribute or attributes possessed in common by all animals; the concept 'metal' is the notion corresponding to the collection of attributes which is found in all metals.

A judgment is the product of the comparison of two concepts and may be considered subjectively or objectively. Regarded subjectively, that is, as an act of the mind, it is a recognition of a certain relation (agreement or disagreement, according to some logicians) between two notions or concepts. In the judgment 'man is mortal,' for example, there are two concepts, 'man' and 'mortal,' and there is a recognition of a certain relation (agree

ment) between them. In the judgment 'no man is perfect,' there are two concepts, 'man' and 'perfect,' and a recognition of a certain relation (disagreement) between them. Similarly, in the judgments 'all metals are elements,' 'all sensations are feelings,' 'all material bodies are extended,' 'matter gravitates,' there are two concepts, and a recognition of a certain relation between them.

It is evident that our definition of concept or of judgment does not include any concepts or judgments that are intuitive, or as they are called *à priori*, that is, not the result of experience, but due to the very nature, constitution, or original forms of the mind. Logic, as defined above, does not inquire into the truth or falsity of these *à priori* concepts and judgments, the existence of which is affirmed by some and denied by others. It does not lay down the conditions to which these must conform in order that they may be true. It treats of the principles and conditions to which those concepts and judgments which are products of comparison must conform in order that they may be free from error and self-contradiction.

A reasoning is the act of the mind by which it passes from one or more judgments to another contained in or warranted by them. It is the recognition of a certain relation between two or more judgments. In the simplest form of reasoning, that is, in immediate inference, a judgment is inferred from another judgment, while in the most complex form, in induction, for instance, a judgment is the result of the comparison of a number of judgments. In the inference "All men are mortal, therefore no man is immortal," we have an example of the former. In the inference "John is dead, James is dead, all men of past ages have died; therefore, all men now living will die, or all men are mortal," we have an example of the latter. In another form of reasoning called Syllogistic, a judgment is the result of the comparison of two judgments; that is, a relation between two concepts is established by comparing each with a third. In the reasoning "All men are fallible, philosophers are men; therefore philosophers are fallible," there are the three concepts, 'philosophers,'

'man,' and 'fallible,' and a relation between the first and the last is established by means of the second. In the first judgment, there is the recognition of a relation between the two concepts 'man' and 'fallible.' In the second, between 'philosophers' and 'man.' In the third between 'philosophers' and 'fallible,' as the result of a comparison of the first two judgments.

§ 2. Regarded objectively, that is, as something existing in things or objects, a concept is an attribute or a collection of attributes in which a number of individual things or objects agree[1]. For example, the concept 'man' viewed objectively,

[1] With reference to this passage, Mr Keynes, reviewing this work in *Mind* for October, 1884, has remarked that it "involves a confusion of phraseology if nothing more," and that "it is calculated to suggest to the student a metaphysical doctrine which it is hardly probable that the author himself holds." There is, I maintain, no confusion of phraseology; but there is a change in the meaning of the word *concept* necessitated by a change in the meaning of the term *Logic*. If Logic is an *objective science* "formulating the most general laws of correlation among existences considered as objective," and if the term *concept* is to be retained in that science, a concept must be something existing in things or objects. The concept, like the science itself, must be *objective;* and what is an *objective concept?* I hold that it must be an attribute or collection of attributes in which a number of individual things agree. Nor is the change in the meaning of the word *concept* so great as I have admitted. Mansel, for instance, defines a concept "as a collection of attributes united by a sign, and representing a possible object of intuition." The second charge brought against the passage is that "it is calculated to suggest to the student a metaphysical doctrine which it is hardly probable that the writer himself holds." I suppose that the metaphysical doctrine here alluded to is the Hegelian doctrine of the Identity of Thought and Being or of Logic and Metaphysics. If this doctrine is suggested by that passage, this is not due to any accident but to great correspondence or resemblance between the Logic of Hegel and the Objective Logic of English Logicians. See Appendix E, "The Nature and Province of Objective Logic."

that is, as something existing in men, is the aggregate of attributes in which all individual men agree. Similarly, the concept 'triangle' is objectively the attribute of 'being bounded by three lines'; the concept 'flower' the attribute or collection of attributes in which all individual flowers agree. Thus every concept is *objectively* an attribute or a collection of attributes, and *subjectively* an idea or notion corresponding to that attribute or collection of attributes.

A judgment, regarded objectively, is, according to some writers, a relation between two attributes; according to others, a relation between two things; and according to others again, a relation between a thing and an attribute. For example, the judgment 'all men are mortal,' objectively regarded, has been variously considered as a relation between the attribute 'mortality' and the collection of attributes 'humanity,' between the two groups of things 'all men' and 'mortal,' and between the group of things 'all men' and the attribute 'mortality'; that is, in that judgment the attribute 'mortality' coexists with the attribute 'humanity,' or, the group of things called 'mortal' includes the group of things called 'man,' or, the attribute 'mortality' belongs to the group of things called 'man.' The judgment 'all metals are elements' is a relation existing between two collections of attributes, namely, those of 'metal' and of 'element,' or between two groups of things, namely, 'metals' and 'elements.' Similarly, every judgment, objectively regarded, is a certain relation between things and attributes.

A reasoning, objectively regarded, is the inference of a relation between two things or attributes from one or more given relations of things and attributes. For example, in the reasoning "All men are mortal, kings are men; therefore, kings are mortal," a relation between 'kings' and 'mortal' is inferred from two given relations between things, namely, (1) a relation between 'men' and 'mortal' expressed in the first judgment, and (2) a relation between 'kings' and 'men' expressed in the second judgment. Similarly, in all reasonings, objectively regarded, a relation universal or particular between two things or attributes

or between a thing and an attribute is inferred from one or more given relations of things and attributes.

From this direct and close connexion between thought, and attributes and things, or, between concepts, judgments, reasonings, on the one hand, and attributes, relations of attributes and things, and inferences, on the other, Logic may be regarded (from the objective point of view) as the science of the most universal relations and correlations of things and attributes, that is, the science of the principles and laws to which we must conform in order that a relation established by comparison of things and attributes, or inferred from one or more given relations between them, may be true.

§ 3. A concept is expressed in language by a single word, or a combination of words, called a term or name. For example, the concept 'man,' or, the aggregate of attributes in which all men agree as well as the idea or notion corresponding to it, is signified or expressed by the word *man*. The concepts 'metal,' 'flower,' 'animal,' 'horse,' that is, both the aggregates of attributes and the ideas corresponding to them, are expressed by those words, respectively. Similarly, the combinations of words 'good man,' 'elementary substance,' 'red flower,' 'round table,' are names or symbols for certain concepts.

A judgment is expressed in language in the form of a sentence, called a proposition. For example, the judgment explained above as expressing a relation between the two concepts 'man' and 'mortal' is expressed in the sentence 'man is mortal.' A reasoning is expressed in language in a series of connected sentences called, an argument. The reasoning explained above as establishing a relation between the two concepts 'philosopher' and 'fallible' by means of a third concept 'man' is expressed in the argument "All men are fallible, philosophers are men; therefore, philosophers are fallible."

From the direct and close connexion between thought and language, between concepts, judgments and reasonings on the one hand, and words and sentences, or names, propositions and arguments on the other, Logic has been regarded as conversant

about language, as the science of the use of names, propositions, and arguments, that is, the science of the principles and rules to which we must conform in order that we may be right and free from fallacy and self-contradiction in the use of names, propositions, and arguments.

Logic has been thus defined from three distinct points of view. The first definition we have given above is from the psychological or subjective point of view, the second from the objective point of view, and the third or last from the linguistic point of view. These definitions reveal also the relations of Logic to the other sciences according as it is regarded from one or other of these three stand-points. The first places it among the mental sciences, and makes it dependent upon the psychology of cognition. The second places it among the objective sciences, and makes it the most general of all sciences, treating of those principles and laws which are equally true of all phenomena and things, both mental and material. The third places it among the linguistic or philological sciences, and makes it dependent upon grammar and language generally. On the first view, Logic treats of the processes and products of conception, judgment, and reasoning. On the second, it treats of things in their universal aspects and relations, that is, of the most general aspects of things, of their fundamental relations, and of relations between relations; on the third, it treats of language, that is, of the use of names, propositions and arguments, or rather of words and sentences.

§ 4. Most logicians have adopted one or other of these views to the exclusion of the other two. A philosopher of mind will naturally adopt the first view and its appropriate phraseology. A scientific man will adopt the second and its appropriate phraseology; while a practical man, with a knowledge of mental philosophy as well as of physical science, will try to combine the first or the third with the second. He will adopt the phraseology of either of the former, but constantly refer to the second for its real meaning, signification, or import. The third view cannot really be held by itself, and though Whately

seems to have maintained it from what he says in many parts of his '*Elements*[1]', nevertheless what he really meant is, that Logic does not treat of reasoning apart from, but only as expressed in, language. "If any process of reasoning," says he, "can take place in the mind without any employment of language, orally or mentally, such a process does not come within the province of the science here treated of[2]." Whately really adopted the subject-matter of the first view, and only the phraseology of the third. This is also evident from his definition of Logic 'as the science and also as the art of reasoning.'

§ 5. Hamilton adopts the first view, and defines Logic as "the science of the laws of thought as thought, or the science of the formal laws of thought, or the science of the laws of the form of thought[3]," that is, as the science of those universal laws or principles to which thought must conform in order that its products, *viz.*, concepts, judgments, and reasonings, may be valid. Hamilton uses the word *valid* to mean free from inconsistency or self-contradiction, and by laws of thought he means only the fundamental principles of consistency, that is (1) the Principle of Identity, (2) the Principle of Contradiction, and (3) the Principle of Excluded Middle. The first means that A is A, that a thing is what it is, that while 'A' is 'A,' it cannot be anything else. The second means that A cannot be both B and not-B, at the same time, in the same place, and in the same respect. If the proposition 'A' is 'B' be true, then the proposition "A is

[1] Whately writes, for example:—"Logic is entirely conversant about language." Again, "It (Logic) is, therefore (when regarded as an art), the art of employing language properly for the purpose of reasoning and of distinguishing what is properly and truly an argument from spurious imitations of it."—*Elements*, 9th Edition, p. 37.

[2] Whately's *Elements*, 9th Edition, p. 37.

[3] *Lectures*, Vol. III. pp. 25, 26. See also pp. 4, 17, 24. On p. 24 Hamilton defines Logic as 'the science of the necessary forms of thought,' and afterwards developes this definition into the expression given in the text. By 'thought as thought' Hamilton means 'the form of thought to the exclusion of the matter' (p. 15).

not-B" cannot be true. If a thing be *red*, it cannot at the *same time* be *not-red.* It may lose redness afterwards, or, it may not be red in all its parts; but if any part of it be red, that same part cannot be *not-red* at the same time. The third means that one or other of two contradictory terms must be true of one and the same thing, the middle or the mean between them being excluded. 'A is either B or not-B'. Here 'B' and 'not-B' are two contradictory terms, and A must be one or other of the two. It cannot be neither. 'This thing is either red or not-red'; this proposition means that the thing must be one or the other, 'red' or 'not-red'—*i. e.* if not 'not-red' then 'red'; and if not 'red,' then 'not-red.' It cannot be anything else than either 'red' or 'not-red.' The two concepts, 'red' and 'not-red' cover the whole sphere of thought and existence; and every possible as well as real object must be one or the other. It is evident that the concept 'not-red' is so indefinite, that it, in fact, includes every thing real or imaginary except 'red'.

According to Hamilton, if a thought does not violate any of the above three laws of thought, then it is *valid;* and the science of Logic is entirely conversant about the forms or the uniform and constant modes of thinking in conformity to those laws, to the entire exclusion of the matter of thought. He does not require that the products of thought must agree with actual realities; the only condition which they must fulfil, according to him, is that they must be free from self-contradiction or inconsistency.

§ 6. In his *Examination of Hamilton's Philosophy*, Mill adopts the first view with the qualification that the products of thought must not only be *formally* valid, but true or objectively real. He defines Logic as "the art of thinking, which means of correct thinking, and the science of the conditions of correct thinking," that is, "the science of the conditions on which right concepts, judgments, and reasonings depend."

The products of thought, according to Mill, should be not only free from inconsistency or self-contradiction, *i. e.*, *valid* in Hamilton's sense, but must also be *true*, *i. e.*, 'agree with the

reality of things.' A concept 'must be a concept of something real, and must agree with the real fact which it endeavours to represent, that is, the collection of attributes composing the concept must really exist in the objects marked by the class-name.' A judgment must be a true judgment, that is, the objects judged of 'must really possess the attributes predicated of them.' A reasoning 'must conduct to a true conclusion[1].'

In the work referred to Mill thus really adopts the subject-matter of the second view, and only the phraseology of the first. The qualification introduced by him into the first view as noticed above has really the effect of changing it into the second[2].

In his *System of Logic* Mill adopts the phraseology of the third view, but always refers to the second for the real import or meaning of his names, propositions, and arguments. He, in fact, holds the second view, and takes the subject-matter of Logic to be what it is according to that view, though in his treatment of the science he freely uses the phraseology of the third[2].

§ 7. Herbert Spencer adopts the second view, and defines Logic as the science which "formulates the most general laws of correlation among existences considered as objective," as the science which "contemplates in its propositions certain connexions predicated, which are necessarily involved with certain other connexions given; regarding all these connexions as existing in the *Non ego*—not it may be, under the form in which we know them, but in some form[3]."

§ 8. We shall not confine ourselves to any of these views. But regarding Logic as primarily or immediately concerned with thought, and, secondarily, or as a means to an end, with language in which thought is expressed, and ultimately with attributes and things, mental or material, real or imaginary, the object-matter of all thought, we shall freely adopt the phraseology of any or all of them, whenever this seems desirable for purposes of explanation and illustration.

[1] Mill's *Examination of Hamilton's Philosophy*, 4th ed. pp. 564, 470.

[2] See Appendix E.

[3] Spencer's *Principles of Psychology*, 2nd ed. Vol. II. p. 87.

§ 9. The relation of Logic to the other sciences is shown in the following tabular views :—

I.

LOGIC.

MATHEMATICS.

Material Sciences.	Mental Sciences.
Physics.	Psychology.
Chemistry.	Logic.
Geology.	Æsthetics.
Botany.	Ethics.
Zoology.	Religion.

Anthropology.

II.

Logic.
Mathematics.
Physics.
Chemistry.
Geology.
Biology ... { Botany. Zoology.
Psychology ... { Logic. Æsthetics. Ethics. Religion.
Sociology.

In the first table the mental and the material sciences are placed in two separate series, and Logic and Mathematics are

placed above both, as their principles are equally applicable to the sciences in the two series. Logic is placed above Mathematics, as it is the most general and abstract of all sciences, as its principles are applicable to Mathematics as well as to the other sciences. In the second table the same relation is shown by placing Logic at the top, and Mathematics next to it. The other sciences are arranged in order of generality, the one lying above being more general than the one lying below. Thus Mathematics is placed above Physics as mathematical principles are applicable to physical phenomena; Physics above Chemistry as physical laws are applicable to chemical phenomena, and so forth. In both the tables, Logic is also placed under Psychology as it treats of the mental processes of conception, judgment and reasoning and of their products, and lays down the laws to which they must conform in order that they may be valid.

§ 10. The end of Logic as defined here is the attainment of truth so far as truth can be obtained by thinking, that is, by the processes of naming, definition, classification, generalization, inference, &c., employed upon the data, or materials, supplied by direct observation, experiment, perception, or intuition. Some logicians (Ueberweg, for example) have indeed made *all* truth the end of Logic, and defined it as "the science of the regulative principles of human knowledge[1]," that is, of all knowledge both intuitive and inferential, immediate and mediate. But, following the British Logicians in general, I have defined Logic so as to exclude intuitive truth from its scope and province. According to Ueberweg, perception and percepts are as much a part of Logic as conception, judgment, and reasoning, while all British Logicians, whatever their differences may be on other points, agree in excluding intuition and intuitive truth from the jurisdiction of Logic[2].

Truth is the agreement of thought with its object, and is said to be either *formal* or *real*. It is *real* when the object of thought actually exists,—is something either material or mental. It is

[1] Ueberweg's *Logic*, English Translation, p. 1.

[2] See Ueberweg's *Logic*, pp. 1, 17, 77, 78; and Mill's *Logic*, Vol. I. pp. 5, 6, 8.

formal when the object, whether actually existing or not, is simply free from any self-contradiction. The latter is the end of what is called Formal Logic, and the former of what is called Material Logic.

In Formal Logic, the concepts, judgments, and reasonings need not be really true. It is sufficient if they conform to the fundamental principles of consistency or laws of thought, as they are called, and be free from any inner contradiction or inconsistency. In Material Logic, also called by Mill the Logic of Truth, they must be true or right, and correspond to the realities actually existing; they must be valid not only formally, but also really; they must be free not only from any self-contradiction, but also from any inconsistency with reality, that is, a concept must be an attribute or a collection of attributes actually existing in things, a judgment, a relation between two true concepts, and a reasoning must lead to a conclusion that agrees with fact.

The end of Material Logic is thus the attainment of truth in the stricter and proper sense, that is, of real truth, while the end of Formal Logic is merely consistency or freedom from self-contradiction.

Formal Logic is often called Pure Logic, and also the Logic of Consistency. Hamilton's definition of Logic, as given above, is a definition of Formal Logic, while Mill's and Spencer's are definitions of Material Logic. In the latter we are concerned with terms, propositions, and arguments that have reference to actual existences, while in the former we are concerned not with what is actual, but with what is possible, not with what is real in Nature, but with what may be realized in Thought. Formal Logic includes in its sphere all possible notions, judgments, and reasonings, or all possible attributes, and their relations, and does not confine itself to what is actual or real in Nature.

The definition which we have given at the beginning of this chapter is that of Formal or of Material Logic according as the word *valid* is taken to mean mere conformity to the principles of consistency, or agreement with reality, that is, according as it means *merely formally valid* or *really valid and true*. If the

products of comparison, namely, concepts, judgments, and reasonings, are required to agree with the actually existing things and phenomena, then our definition becomes the definition of Material Logic. If, on the contrary, they are required simply to be free from self-contradiction, then our definition becomes the definition of Formal Logic.

§ 11. Logic is usually regarded as consisting of three parts,—the first part treating of the process and products of conception; the second, of judgment; and the third, of reasoning or inference. To these three parts may be added a fourth, namely, Method, treating of the arrangement or disposing of a series of reasonings in an essay or discourse. Method has been defined as "the art of disposing well a series of many thoughts, either for discovering truth when we are ignorant of it, or for proving it to others when it is already known." "Thus there are two kinds of Method, one for discovering truth, which is called analysis, or the method of resolution, and which may also be termed the method of invention; and the other for explaining it to others when we have found it, which is called synthesis, or the method of composition, and which may be also called the method of doctrine[1]."

"Without stepping," says Professor Robertson, "beyond the bounds of Logic conceived as a formal doctrine, a fourth department under the name of method or disposing may be added to the three departments regularly assigned—conceiving (simple apprehension), judging, reasoning; and this would consider how reasonings, when employed continuously upon any matter whatever, should be set forth to produce their combined effect upon the mind. The question is formal, being one of mere exposition, and concerns the teacher in relation to the learner. How should results, attained by continuous reasoning, be set before the mind of a learner? Upon a line representing the course by which they were actually wrought out, or always in the fixed order of following from express principles to which preliminary assent is required? If the latter, all teaching becomes synthetic, and

[1] Professor Baynes' *Port Royal Logic*, pp. 308—9.

follows a progressive route from principles to conclusions, even when discovery (supposing discovery foregone) was made by analysis or regression to principles, of which expository method no better illustration could be given than the practice of Euclid in the demonstration of his '*Elements*.' On the other hand, it may be said that the line of discovery is itself the line upon which the truth about any question can best be expounded or understood for the same reason that was found successful in discovery, namely, that the mind (now of the learner) has before it something quite definite and specific to start from; upon which view, the method of exposition should be analytic or regressive to principles, at least wherever the discovery took that route. The blending of both methods, when possible, is doubtless most effective; otherwise it depends upon circumstances—chiefly the character of the learner, but also the nature of the subject in respect of complexity, which should be preferred,—when one alone is followed[1]."

§ 12. By some logicians Deductive Logic is regarded as identical with Formal Logic; by others as a part of Material Logic. According to all, it does not directly concern itself with the real truth or falsity of its data, but with their formal correctness or freedom from inconsistency, and with the legitimacy of the results from them. In this work it is proposed to treat of the following subjects:—The fundamental principles; the name, the concept, the term and its divisions; denotation, connotation, extension, comprehension; the proposition, the judgment, and their divisions; the predicables; the theory of predication and the import of propositions; definition, division; inference, reasoning and their divisions; immediate inference and its divisions; the syllogism, its divisions, its canons, its rules, its figures, its moods, its function and value; reduction; fallacies; probable reasoning and probability.

[1] *Encyclopædia Britannica*, 9th edition, Vol. I. p. 797.

CHAPTER II.

THE FUNDAMENTAL PRINCIPLES OF DEDUCTIVE LOGIC.

§ 1. THERE is great difference of opinion among logicians as to the nature, number, name, origin, and place in a Treatise on Logic, of what we have here called the fundamental principles of Deductive Logic. They may be stated as follows:—

(1) "A is A." "A thing is what it is." "Every thing is equal to itself." "Every thing is what it is." This is called the Principle or Axiom of Identity. It really means that the data, with which we start in Deductive Logic, must remain unaltered; that, by them we must abide in all our deductions and reasonings. If we have granted or assumed that a certain thing possesses a certain attribute, we must always admit that; if we have used a term in a certain meaning, we must always use it in that meaning, or give notice when any change is made. In Deductive Logic things and their attributes, or thoughts, are supposed to be unalterably fixed; and the same thing must always be regarded as possessed of the same attributes. In nature, no doubt, a thing may change and have attributes which it did not originally possess; but Deductive Logic takes no cognizance of such changes. It assumes, on the contrary, that all things and their relations are as absolutely fixed and permanent as are the properties and relations of Geometrical Figures. And the principle or axiom of identity expresses this unalterable or absolutely fixed nature of things, postulated in Deductive Logic, by stating that "Every thing is what it is," that is, it cannot change and be other than what it is, nor can

it lose any of its properties or attributes. In other words, the element of time or change has no place in Deductive Logic.

§ 2. (2) "A cannot be both B and not-B." "The same thing cannot be both B and not-B." "This paper cannot be both white and not-white." This is called the Principle or Axiom of Contradiction. It means that two contradictory terms B and not-B cannot both be true, at the same time, of one and the same individual thing A. If the term B be true of the individual thing A, then the term not-B cannot, at the same time, be true of it; or if the term not-B be true of it, then B cannot, at the same time, be true of it. In other words, two contradictory propositions cannot both be true; taking A to mean an individual thing, one and the same thing, and using B in the same sense in both, the two propositions 'A is B' and 'A is not-B' are contradictory, and cannot both be true: if one be true, the other must be false; that is, if 'A is B' be true, then 'A is not-B' must be false; and if 'A is not-B' be true, then 'A is B' must be false. For example, a leaf cannot, at the same time, be 'green' and 'not-green'; if it is 'green,' it cannot, at the same time, be 'not-green' (see p. 10); a piece of gold cannot, at the same time, be 'yellow' and 'not-yellow'; if it is 'yellow,' it cannot, at the same time, be 'not-yellow'; a sample of water cannot, at the same time, be 'liquid' and 'not-liquid,' 'cold' and 'not-cold,' 'hot' and 'not-hot'; if it has one quality, it cannot, at the same time, have the contradictory quality; 'cold' and 'not-cold,' 'liquid' and 'not-liquid' are contradictory qualities, and cannot be possessed, at the same time, by the same thing. Similarly, a thing cannot at the same time be 'mortal' and 'not-mortal,' 'extended' and 'not-extended,' 'organized' and 'not-organized,' 'existent' and 'not-existent,' 'good' and 'not-good'; if it has one of these contradictory attributes, it cannot, at the same time, have the other.

§ 3. (3) "A is either B or not-B." "The same thing is either B or not-B." "This paper is either white or not-white." This is called the Principle or Axiom of Excluded Middle. It means that two contradictory terms, B and not-B, cannot both

be false, at the same time, of one and the same individual thing. If the term B be not true of the individual thing Ω, then the term not-B must be true of it; if the term not-B be not true of it, then B must be true of it. In other words, two contradictory propositions cannot both be false; taking A as before to mean one and the same individual thing, and using the term B in the same sense in both, the two propositions 'A is B' and 'A is not-B' are contradictory and cannot both be false; if one be false, the other must be true; that is, if the proposition 'A is B' be false, then the proposition 'A is not-B' must be true, and if 'A is not-B' be false, then 'A is B' must be true. For example, the two propositions, 'a leaf is green,' and 'a leaf is not-green,' cannot both be false; a leaf is either 'green' or 'not-green': if the term 'green' be not true of a leaf, then its contradictory 'not-green' must be true of it; that is, two contradictory terms cannot both be false of one and the same thing. Similarly, 'yellow' and 'not-yellow,' 'liquid' and 'not-liquid,' 'good and not-good' cannot both be false of one and the same thing, such as a piece of gold, a sample of water, or any other individual thing: if one of them be false of any one of these things, then the other must be true of it. In other words, of the two contradictory propositions "a leaf is green" and "a leaf is not-green," both cannot be false; if one be false, the other must be true; similarly, of the contradictory propositions "this sample of water is "cold," and "this sample of water is not-cold," "this piece of gold is yellow," and "this piece of gold is not-yellow," "this piece of chalk is solid," and "this piece of chalk is not-solid," both cannot be false: if one be false, the other must be true.

According to the Principle of Contradiction, two contradictory propositions cannot both be true, that is, one must be false; and, according to the Principle of Excluded Middle, both of them cannot be false, that is, one must be true. Of the two contradictory propositions, 'A is B' and 'A is not-B' (taking A to mean an individual thing, and using A and B in the same sense in both), one must be false according to the former, and

one must be true according to the latter; that is, if the proposition 'A is B' be true, then the proposition 'A is not-B' must be false; if 'A is not-B' be true, then 'A is B' must be false; and if the proposition 'A is B' be false, then 'A is not-B' must be true; if 'A is not-B' be false, then 'A is B' must be true. According to the two principles, therefore, the truth of one contradictory proposition implies the falsity of the other, and the falsity of one implies the truth of the other; that is, of two contradictory propositions one must be true by the Principle of Excluded Middle, and the other must be false by the Principle of Contradiction.

We have taken above A to mean an individual thing, one and the same thing; and, in that case, two contradictory terms B and not-B cannot both be either true or false of A; or, in other words, the two propositions 'A is B' and 'A is not-B' are contradictory, and cannot both be either true or false. But if A signifies a class of things, that is, if A be a general term or a name for each individual of a number of things, then the two contradictory terms B and not-B might both be true or false of A. 'B' might be true of some individuals and false of others, all belonging to 'A,' so that the two propositions 'A is B' and 'A is not-B' would both be false in one sense, and true in another—*false* if 'A' is taken universally, that is, if A stands for all the individuals of the class, and *true* if 'A' is taken partially, that is, if A stands for a part, or at least one individual, of the class. Let us take, for example, the common name 'man' and the two contradictory terms 'wise' and 'not-wise.' Now, man as a class is not either 'wise' or 'not-wise'; in other words, the two propositions 'man is wise' and 'man is not-wise' are both false, if the term '*man*' be taken universally to denote all men, while they are both true if the term 'man' be taken partially to denote some men or at least one man. Hence two contradictory terms may be both false of a class; that is, the two propositions 'A is B' and 'A is not-B' may be both false, if 'A' be a general term or common name. In other words, the two contradictory propositions are then not 'A is B' and 'A is not-B,' but 'all A is B,'

and 'some A is not B'; and of these, both can be neither true (Law of Contradiction), nor false (Law of Excluded Middle); one must be false, and the other true. If all the things belonging to the class A are, however, individually considered, that is, if 'A' be taken as standing, at the same time, for a single individual only, then, of that individual, either 'B' or 'not-B' must be true. Thus 'wise' or 'not-wise' must be true of a single individual man, that is, of every man considered as an individual thing, one or other of these two contradictory terms must be true, though, on the whole, some individuals may belong to the class of wise, and others to the class of not-wise.

§ 4. (4) The next principle that we shall give here is a postulate of Logic. It is thus stated by Hamilton:—"The only postulate of Logic which requires an articulate enouncement is the demand, that before dealing with a judgment or reasoning expressed in language, the import of its terms should be fully understood; in other words, Logic postulates to be allowed to state explicitly in language all that is implicitly contained in the thought[1]:" that is, given a term, proposition, or argument, the thought expressed by it, or its meaning and import may be stated in any other form of words, which expresses the same thing. Thus, in describing the logical characters of a term or of a proposition, it is allowable to make any verbal changes we like, in order to reduce it to the logical form, provided the meaning remains the same. In testing an argument we may state it in any form of words we please, provided the thought contained in the constituent propositions or in the argument as a whole remains unaltered.

§ 5. Mill regards all the four principles given above as postulates. "Whatever is true in one form of words is true also in every other form of words which conveys the same meaning[2]." He gives this for the Principle of Identity, regards it as the most universal postulate of Logic, and calls it a first Principle of

[1] Hamilton's *Lectures*, Vol. III. p. 114.

[2] *An Examination of Hamilton's Philosophy*, p. 482.

Thought. According to him the postulate we have given above is included in this. For the Principle of Contradiction, Mill gives the following postulate: "The affirmation of any assertion and the denial of its contradictory are logical equivalents, which it is allowable and indispensable to make use of as mutually convertible[1]." For the affirmation of the assertion "A is B," we may substitute the denial of its contradictory "A is not B"; or for the affirmation of the assertion "A is not B" we may substitute the denial of its contradictory 'A is B': that is, the denial of 'A is B' and the assertion of its contradictory 'A is not B' are logically the same. For the Principle of Excluded Middle, Mill gives the postulate that it is allowable "to substitute for the denial of either of two contradictory propositions, the assertion of the other[2]." That is, of the two propositions 'A is B' and 'A is not B,' we may substitute the assertion of one for the denial of the other: for the denial of 'A is B' we may substitute the assertion of 'A is not B'; and for that of the latter the assertion of the former.

Mill calls his three postulates the 'universal postulates of reasoning,' which ought to be placed, at the earliest, in the second part of Logic—the Theory of Judgments; since they essentially involve the ideas of truth and falsity, which are attributes of judgments only, not of names or concepts. This remark seems not applicable to his first postulate (that for the Law of Identity: "Whatever is true in one form of words is true also in every other form of words, which conveys the same meaning") as we require it for making verbal alterations, and for stating in logical form the meaning of a term, before describing its logical characters. Still less is the remark applicable to the postulate which we have given above. We require the aid of that postulate in order to state explicitly the thought that is implicitly contained in a term, and, in the case of an ambiguous term, to recognize its different meanings and treat them as such. It is hardly necessary to say that it is impossible to describe the logical characters

[1] *Ibid.* p. 488. [2] *Ibid.* p. 490.

of a term without fully understanding and explicitly stating its meaning or meanings, the thought or thoughts, the attribute or thing, signified by it. For this reason, all the principles are here placed in the Introduction before the first part of Logic treating of Terms or Concepts.

Hamilton calls the first three principles the 'fundamental laws of thought,' and prefers to call the second the 'Law of Non-contradiction,' "as it enjoins the absence of contradiction as an indispensable condition of thought[1]."

Ueberweg calls them the Principles or Axioms of Inference, and places them at the beginning of the part treating of Inferences. To these three he adds a fourth, namely, the Axiom of the (determining or sufficient) Reason. The statement of this Principle or Axiom by Leibnitz seems to be the best, and is as follows :—" In virtue of this principle we know that no fact can be found real, no proposition true, without a sufficient reason, why it is in this way rather than in another."

According to Ueberweg the Axiom of Contradiction and the Axiom of Excluded Middle may be comprehended in a general principle, namely, the Principle of Contradictory Disjunction. The formula of this is :—'A is either B or is not-B,' which means that 'A' cannot be *both* 'B' and 'not-B' (Law of Contradiction), and that it must be *one* or the *other* (Law of Excluded Middle).

Ueberweg gives also another axiom which he calls the Axiom of Consistency. He states it as follows :—'A which is B is B, *i. e.*, every attribute which belongs to the subject notion may serve as a predicate to the same.' He regards this axiom as allied with the Axiom of Identity[2].

§ 6. To the principles given above should be added the following :—

(5) Aristotle's *Dictum de omni et nullo*[3]. "Whatever is affirmed or denied of a class distributively may be affirmed or

[1] Hamilton's *Lectures*, Vol. III. p. 82.

[2] Ueberweg's *Logic*, English Translation, pp. 231, 275, 281, 283, &c.

[3] See below, Part III. Chapter IV.

denied of every thing belonging to that class"; or, "what belongs to a higher class belongs to a lower." Some logicians maintain that it can be deduced from the three Laws of Thought, while others regard it as an independent axiom incapable of deduction from those laws.

(6) The fundamental axioms or canons of Syllogism as given by different logicians (Mill, Martineau, Thompson, Lambert, Whately, &c.[1]).

(7) The Mathematical Axioms :—(1) that of *Argumentum à fortiori*, namely, that "a thing which is greater than a second, which is greater than a third, is greater than the third"; (2) the axiom that "two things equal to the same thing are equal to each other"; and other axioms of a similar nature.

[1] See below, Appendix A.

PART I.—TERMS.

CHAPTER I.

The Various Divisions of Terms.

§ 1. A *name* may be defined as a sign for a thing or things. More accurately, it is a word, or a combination of words, signifying some object of thought,— something real or imaginary, mental or material, substantive or attributive, phenomenal or nöumenal. For example, the words 'animal,' 'plant,' 'flower,' 'table,' 'paper,' 'chair' are names of real things, while the words 'centaur,' 'golden mountain,' &c., are names standing for imaginary objects; the words 'mind,' 'soul,' 'spirit,' 'self,' &c., are names signifying mental things or substances, while the words 'gold,' 'silver,' 'mineral,' 'copper,' &c., are names standing for material things; the words 'sensation,' 'pleasure,' 'pain,' 'perception,' 'imagination,' 'memory,' &c., are names expressing attributes of mind, while 'solidity,' 'colour,' 'figure,' 'hardness,' &c., are words signifying attributes of matter; the words 'thinking,' 'perceiving,' 'feeling,' 'wishing,' 'hoping,' &c., are names expressing acts or phenomena of mind, while the words 'moving,' 'melting,' 'expanding,' 'cooling,' &c., are words signifying phenomena or changes of bodies; the words 'thing-in-itself,' 'matter-in-itself,' 'mind-in-itself,' are names expressing nöumena or realities which are believed to underlie all phenomena; and the

words 'substance,' 'substratum,' those in which attributes are supposed to be inherent.

- NAMES (of things in the widest sense).
  - Names (of real things).
    - Names (of mental things).
      - I
        - Names (of substances).
        - Names (of attributes).
      - II
        - Names (of noumena).
        - Names (of phenomena).
    - Names (of material things).
      - I
        - Names (of substances).
        - Names (of attributes).
      - II
        - Names (of noumena).
        - Names (of phenomena)
  - Names (of imaginary things).

A concept is the product of the comparison of two or more individual things and may be viewed subjectively or objectively. Regarded objectively, it is an attribute, or a collection of attributes, which is possessed in common by a number of individual things; and, regarded subjectively, it is an idea or notion corresponding to that attribute or collection of attributes. It is signified or expressed in language by a word, or combination of words, called a name or term. It may be symbolized by any sign whatever. For example, any letter of the alphabet, or any other sign, may be made to stand for any concept. *Conception* usually means the process of forming concepts; but it is sometimes used for the product also, and is, then, taken by some logicians to signify an idea corresponding to an individual thing as well as an idea corresponding to an attribute or a group of attributes common to a number of individuals. The former is called by them an *individual* conception, and the latter a *general* conception, or *notion*. For example, an idea corresponding to an individual thing, such as a particular man, animal, tree, or flower, is an individual conception, while an idea corresponding to an attribute or collection of

attributes possessed in common by a number of individual things, such as men, animals, trees, or flowers, is a general conception. Objectively regarded, an individual conception is an individual thing itself, while, subjectively, it is an idea of the thing.

The process of forming concepts may be regarded as consisting of the following steps: (1) the observation of individuals; (2) the analysis of each of them into its constituent attributes; (3) the comparison of them with one another, in order to find out the attributes in which they all agree, and to separate these from those in which they differ; (4) the mental unification, if possible, of these common attributes, that is, the thinking of them together or the making of the aggregate of them a single object of thought; (5) the expression or symbolization of this aggregate, or single object of thought, by an audible, visible, or other sign, usually by a word or combination of words, called a name or term . For example, in forming the concept 'metal,' (1) different individual metals, such as gold, silver, copper, mercury, platinum, &c., must be observed and experimented upon; (2) the attributes of each of them must be found out by physical and chemical methods; (3) they must be compared with one another in order to find out the attributes in which they agree; (4) these attributes, when found out, must be thought of together; and (5) symbolized for reference afterwards as well as for communication to others, by a word, or some other sign. The concepts 'man,' 'horse,' 'plant,' 'animal,' 'book,' 'table,' 'element,' 'flower,' &c., are formed in the same manner.

A *term*, in the wider sense, is a name. It is the expression in language of a concept or of an individual or individuals. In the narrower sense, it is the subject or the predicate of a proposition, that is, that of which something is said, or that which is said about something, in a sentence or proposition. For example, the words 'man,' 'horse,' 'plant,' 'flower,' and the combinations of words 'flowering plant,' 'elementary substance,' 'elements that conduct heat and electricity,' 'animals that live in water,' 'the smell of a flower,' are terms in the wider sense, but not in the narrower sense, in which they must be either the subject or

the predicate in a proposition, that is, either they must be affirmed or denied of something, or something must be affirmed or denied of them; in other words, a term, in the narrower sense, is a part of a sentence, while, in the wider sense, it is a name, whether part of a sentence or not. Every term or name, though it may not actually form, is capable of forming either the subject or the predicate of a proposition, that is, something may be affirmed or denied of it, or it may be affirmed or denied of something; and this is the best test by which a term or name may be distinguished from a mere word or combination of words. Terms are divided by logicians into certain broad divisions, which are given below in a tabular form :—

TERMS ...	I.	Single-worded, *e.g.*, man.
		Many-worded, *e.g.*, man of business.
	II.	Singular, *e.g.*, Socrates, the sun.
		General, *e.g.*, book.
		Collective, *e.g.*, a library.
	III.	Concrete, *e.g.*, man, book.
		Abstract, *e.g.*, redness.
	IV.	Positive, *e.g.*, water.
		Negative, *e.g.*, inorganic.
		Privative, *e.g.*, blind.
	V.	Correlative, *e.g.*, husband and wife.
		Absolute, *e.g.*, metal, God.
	VI.	Connotative, *e.g.*, man.
		Non-connotative, *e.g.*, squareness.

§ 2. The first division of terms is into single-worded and many-worded. A *single-worded* term consists of a single word, while a many-worded term consists of a combination of words. For example, the terms 'man,' 'metal,' 'animal,' 'paper,' are single-worded; while the terms 'wise man,' 'rational animal,' 'white paper,' 'yellow flower,' are many-worded. A *many-worded* term may consist of any number of words from two upwards. It may consist of nearly the whole of a sentence or paragraph, provided that it expresses some object of thought, or something of which something may be affirmed or denied, or which may be

affirmed or denied of something. Every term is a word or consists of words, but every word is not a term. A word, or combination of words, which is capable of being employed by itself as a term, is called categorematic, while a word, or combination of words, which must be joined with other words in order to form a term, is called syncategorematic: thus substantives, adjectives, and verbs are categorematic, while all prepositions, articles, conjunctions, interjections, adverbs, &c., are syncategorematic. For example, the words 'man,' 'animal,' 'rational,' 'running,' 'whiteness,' &c., and the combinations of words 'a good man,' 'a rational animal,' 'a flowering plant,' &c., are categorematic, while the words 'and,' 'but,' 'of,' 'when,' &c., and the combinations of words 'instead of,' 'with reference to,' 'on the subject of,' 'very sincerely,' &c., are syncategorematic. It should be observed that the distinction of categorematic and syncategorematic is applicable to the words and combinations of words, while the distinction of single-worded and many-worded is applicable to terms, that is, to those words and combinations of words which are categorematic.

§ 3. The second division of terms is into singular and general. A *singular* term is a name of an individual thing, that is, a name which is applicable, in the same sense, to one thing. For example, the terms 'the present Emperor of Germany,' 'the Metropolis of India,' 'the Ganges,' 'the sun,' 'the moon,' 'Socrates,' 'Plato,' 'the 76th Regiment of Foot in the British Army,' are all singular, signifying each an individual thing or object of thought. A *general* term is a name of each of two or more individual things, that is, a name which is applicable, in the same sense, to each of an indefinite number of things. For example, the terms 'man,' 'flower,' 'animal,' 'metal,' 'element,' 'sensation,' 'state,' 'body,' 'idea,' 'feeling,' are general, standing each for every one of an indefinite number of individual things or phenomena; the term 'man' is a name for every individual of a large class or group of things called men; the term 'flower' is applicable to every individual of a group of things; the term 'feeling' is applicable to each of a large number of mental phenomena; the terms 'idea,' 'thought,' 'hope,' 'joy,' 'sorrow,' are likewise applicable each to

every one of a group of mental things or phenomena. Thus, every general term is a name of each individual of a number of things or phenomena, material or mental.

A general term should be distinguished from a collective term, which is a name for a group of things taken together, and regarded as one—as a single object of thought. Thus, while a general term is applicable to each of a number of things, a collective term cannot be applied to each individual of a multitude separately, but only to all taken together. Thus, 'a library,' 'a regiment,' 'a nation,' 'a forest,' are collective terms: each of them is a name of a collection of many things, taken together, and regarded as one complex whole. The term 'a library,' for example, signifies a large collection of books, and is applicable to all of them collectively, not to any one of them separately; 'a regiment' is a term applicable to a multitude of soldiers collectively, not to any one of them individually. It should be noticed that such collective terms as 'regiment,' 'library,' &c., are general and not singular; the term 'library' is general, inasmuch as it is applicable to any one of the numerous libraries throughout the world; the term 'forest' is likewise general, being applicable to any forest in any country; similarly, the terms 'nation,' 'army,' 'multitude,' 'a few,' 'a crowd,' are both collective and general—*collective*, because each of them is applicable to a number of things taken together and regarded as a whole; and *general*, because it is applicable to each of an indefinite number of such wholes. On the other hand, such collective terms as 'the 76th Regiment of Foot in the British Army,' 'the British Museum,' 'the Bodleian Library,' 'the University College Library,' 'the English people,' are singular, and not general, inasmuch as each of them is applicable to a single collection or complex whole, and not to more than one. Some logicians regard 'regiment' as general, and 'a regiment' as collective; 'nation' as general, and 'a nation' as collective, that is, according to them, a collective term denotes indefinitely an individual collection of things or objects, and this should be expressed by the indefinite article prefixed to it. This distinction in language between a collective

and a general term appears to be good on more than one ground, and should not be overlooked[1].

It should be observed that a general term is applicable to a number of things, not arbitrarily, but in virtue of their agreement in an attribute or collection of attributes. It implies that the things to which it is applicable agree in an attribute or attributes. It is, in fact, a name of a concept as well as of individual things. In technical language it is said to *denote* or signify directly the things to which it is applicable, and *connote*, imply or signify indirectly the attribute or attributes in which they all agree. In other words, a general term is a name of a class, and connotes the attribute or attributes which characterise it, and denotes the individuals which belong to it.

§ 4. The third division of terms is into concrete and abstract. An *abstract* term is a name of an attribute, or a collection of attributes, apart from the substance in which it exists. The word *attribute* is here used in its widest sense to mean any quality, property, or accident of a substance or thing, and, also, any relation of things and qualities. For example, 'animality,' 'humanity,' 'whiteness,' 'triangularity,' &c., are all abstract terms, each signifying an attribute or a group of attributes apart from the substances in which it exists. 'Equality,' 'succession,' 'coexistence' are abstract terms, each signifying a relation of things apart from the things. A *concrete* term is, on the other hand, a name of a substance, or a class of substances. The word *substance* is here used to mean an individual thing mental or material. For example, 'Socrates,' 'the sun,' 'the earth,' 'the table,' 'man,' 'animal,' 'plant,' &c., are all concrete terms, signifying individual things or substances, and not merely attributes. The term 'man' is concrete, inasmuch as it is a name of many things and not merely of the attribute 'humanity' possessed in common by all individual men. For the same reason, adjectives are generally concrete, inasmuch as they are names of things and not merely significant of attributes: the adjective

[1] See Hamilton's *Lectures*, Vol. II. pp. 281—2.

'white,' for example, is a name of all things whatever having the colour 'whiteness,'—a name not merely of this quality, but of every white object. From this it is also evident that adjectives are general and not singular terms.

All adjectives are regarded by Mill and Jevons as concrete and general, that is, as names denoting or signifying directly things and connoting or implying attributes; but it is evident that some of them may signify attributes, and imply attributes of those attributes, and be thus general and abstract, and, also, that they may, in some cases, express attributes only, and be thus abstract or *attributive*. For an adjective may be applied to an attribute as well as to a concrete thing, that is, it may qualify both abstract and substantive nouns. For example, the adjective 'great' may qualify the abstract terms 'goodness,' 'boldness,' 'beauty,' 'generosity,' 'size,' 'extension,' 'firmness,' 'strength,' &c., as well as the concrete terms 'man,' 'philosopher,' 'poet,' 'picture,' &c.; the adjectives 'small,' 'equal,' 'greater,' 'large,' 'less,' &c., may likewise qualify attributes, as well as things; in such cases, adjectives should be regarded as general, and abstract rather than concrete. And, when an adjective is affirmed of a thing, or of an attribute, it suggests to the mind an attribute, and not any thing; for example in the proposition 'snow is white,' the word *white* suggests simply the attribute whiteness; and not any thing or class of things; in the proposition 'gold is yellow,' the adjective *yellow* suggests simply the attribute 'yellowness'; in such cases adjectives are significant of attributes only, and not of things. This is, however, a matter in which logicians differ,—some (Mill, Jevons, &c.[1]) maintaining that all adjectives are names of things, implying attributes, that is, concrete and general; others (Martineau, Fowler, &c.[2]) holding that they are not names of things, but *attributives*, that is, words which "ex-

[1] See Mill's *Logic*, Vol. I. pp. 25, 31, &c.; and Jevons' *Lessons*, p. 21.

[2] See Martineau's *Essays*, Vol. II. p. 345; and Fowler's *Deductive Logic*, 6th Edition, pp. 13, 18.

press characters or attributes, as such, apart from any objects having them."

Abstract terms are sometimes distinguished into singular and general. A singular abstract term is a name of a definite individual attribute. For example, 'milkwhiteness,' 'visibleness,' 'equality,' 'squareness,' &c., are singular abstract terms, signifying each an attribute perfectly definite and incapable of any division. A general abstract term is a name of each of a group of attributes, that is, a name which can be affirmed, in the same sense, of each of an indefinite number of attributes. For example, the terms 'colour,' 'figure,' 'virtue,' 'pleasure,' 'pain,' &c., are abstract, and, at the same time, general, each of them being applicable to every one of a number of attributes: 'colour' may stand for any variety or shade of colour, red, blue, yellow, indigo, &c.; 'figure,' for any kind of figure, triangle, quadrilateral, &c.; 'virtue,' for any species of it, justice, veracity, temperance, benevolence, &c. Whenever any attribute admits of degree, variety, or species, its name may stand for these, and thus become general. A concrete term is of course singular or general according as it is applicable, in the same sense, to one thing only or to more than one.

Logicians, however, differ in this matter; and I wish, therefore, to note the different opinions which they hold :—

(1) Some Logicians hold that the distinction of singular and general is not applicable to abstract terms; and that abstract terms should be placed in a class apart. Mill indicates this view in one passage. He says "To avoid needless logomachies, the best course would probably be to consider these names as neither general nor individual, and to place them in a class apart[1]." Mr Keynes says, "A still more satisfactory solution however is to consider the distinction of general and singular as not applying to abstract names at all[2]". So far as Mill's passage is concerned, I do not think it carries any weight. All that he says about

[1] *Logic*, 8th Edition, Vol. I. p. 30.
[2] *Formal Logic*, p. 12.

'attribute,' 'relation,' 'quantity,' 'quality,' &c. in the chapter on 'Nameable Things' is opposed to it. In fact, throughout his chapters on 'Names' he recognises the distinction as applicable to abstract terms, and one single statement with 'probably' qualifying it does not certainly carry much of the weight of his opinion.

(2) Some Logicians hold that all abstract terms are singular. —"I should doubt," says Mr Keynes, "if any attribute can, strictly speaking, be conceived as many. An attribute in itself is one and indivisible, and does not admit of numerical distinctions[1]." Mr Monck says, "Abstract terms would seem to be singular when considered logically[2]."

(3) Some Logicians hold that all abstract terms are general in as much as an attribute may be possessed by each of a number of individual things.

(4) Some Logicians hold that abstract terms, like concrete, should be divided into singular and general. Mill says:—"Do abstract names belong to the class of general, or to that of singular names? Some of them are certainly general, I mean those which are names not of one single and definite attribute, but of a class of attributes. Such is the word *colour*, which is a name common to whiteness, redness, &c. Such is even the word *whiteness*, in respect of the different shades of whiteness to which it is applied in common: the word *magnitude* in respect of the various degrees of magnitude and the various dimensions of space; the word *weight* in respect of the various degrees of weight. Such also is the word *attribute* itself, the common name of all particular attributes. But when only one attribute, neither variable in degree nor in kind, is designated by the name; as visibleness; tangibleness; equality; squareness; milkwhiteness; then the name can hardly be considered general; for though it denotes an attribute of many different objects, the attribute itself is always conceived as one, not many[3]." Hamilton says:—"The

[1] *Formal Logic*, p. 11. [2] *Introduction to Logic*, p. 102.
[3] *Logic*, p. 30.

notion of the figure of the desk before me is an abstract idea—an idea that makes part of the total notion of that body, and on which I have concentrated my attention, in order to consider it exclusively. This idea is abstract, but it is at the same time individual; it represents the figure of this particular desk, and not the figure of any other body[1]."

Ueberweg says:—"The general conception (in opposition to the individual conception) is not to be confounded with the abstract (in opposition to the concrete, see § 47). The divisions cross each other. There are concrete and abstract individual conceptions and concrete and abstract general conceptions[2]."

It is evident that the question whether the distinction of singular and general is applicable to abstract terms cannot be satisfactorily solved without stating clearly what is meant by a singular and what by a general term. If a *singular* term is a name applicable to *one object of thought*, and if a *general* term is a name applicable to *each of a number of objects of thought*, then the distinction is certainly applicable to abstract terms: for attributes as well as phenomena and substances may be objects of thought; and an abstract term, like a concrete, may be *a name of one object of thought* or *a name of each of a number of objects of thought.* The abstract terms, for instance, "the figure of the desk before me," "the colour of the rose near me," "the solidity of this stone," as well as 'squareness,' 'equality,' 'visibleness,' &c., are each of them applicable to *one object of thought*—to a single definite individual attribute, while the abstract terms 'relation,' 'quality,' 'quantity,' 'figure,' 'attribute,' 'virtue,' &c., are each of them applicable to *each of a number of objects of thought*, that is, to each of a class of attributes: 'relation,' for example, is a name applicable to any relation whatever,—succession, coexistence, resemblance, difference, &c.; 'quality' is a name applicable to any quality of any object whatever.

[1] *Lectures*, Vol. II. p. 287—8.
[2] *Logic*, p. 127. See also pp. 114—115.

According to some Logicians, abstract terms, when they become general, pass into the class of concrete terms. In other words, there is no absolute distinction according to them, between abstract and concrete terms, between attributes and things. The same term may be abstract from one point of view and concrete from another point of view; and the distinction between abstract and concrete terms is only a relative one. This question can not be satisfactorily solved without stating clearly what is meant by an abstract term and what by a concrete term. The definition of a concrete term as 'the name of a thing,' is of course ambiguous; for the word *thing* may mean either a substance, or a phenomenon, or an attribute possessing another attribute. The definition of an abstract term as 'the name of an attribute' is also ambiguous; for the word *attribute* may mean simply an attribute, or an attribute possessing another attribute, or an attribute of an attribute, apart from the substances or phenomena in which they exist.

Terms expressive of phenomena are usually regarded as concrete. A phenomenon is a changing state of mind or matter. It is a change of a thing, thought of with reference to the thing. It is, in fact, the thing in that particular state of change. The terms, for instance, 'the rising of the sun,' 'the boiling of water,' 'the anger which I felt yesterday,' 'the present state of my mind,' &c. are concrete: 'the rising of the sun' means 'the sun in the state of rising'; 'the boiling of water' means 'water in the state of boiling.' If the changes of things are thought of, or signified, apart from the things, then they really become the attributes of those things. Terms expressive of mere appearances, circumstances, or aspects apart from things, should be regarded as abstract: 'the rising of the sun' would be abstract, if it simply meant the circumstance or aspect of rising apart from the thing 'sun'; 'the boiling of water' would be abstract, if it simply meant the appearance or state of boiling apart from the thing 'water.' But this is a matter on which there may be difference of opinion; and until the terms 'concrete' and 'abstract' are more definitely defined,

I do not think there can be any satisfactory solution of the difficulty.

§ 5. The fourth division of terms is into positive, negative, and privative. A *positive* term signifies the presence of an attribute or a substance; a *negative* term, its absence; a *privative* term signifies the present absence of an attribute and implies the capacity for it. For example, 'man' and 'human' are positive; 'not-man' and 'not-human' are negative; and 'blind,' 'lame,' &c., are privative. The term 'pleasant' is positive, 'not-pleasant' negative, while 'unpleasant' would seem to be positive as signifying not merely the absence of pleasure but the presence of some positive pain; 'convenient,' 'not-convenient,' and 'inçonvenient,' 'moral,' 'not-moral,' and 'immoral' are likewise positive, negative, and positive respectively. 'Organic' is positive and 'inorganic' negative; 'metallic' and 'metal' are positive, while 'non-metallic' and 'non-metal' are negative; 'wise' is positive and 'not-wise' negative, while 'ignorant' might be regarded as negative or privative according to circumstances. It is evident from the examples given above that these terms may be concrete or abstract,—*concrete* when implying the presence or absence of things or substances, and *abstract* when of attributes only.

§ 6. The fifth division of terms is into correlative and absolute. A *correlative* term is a name of an attribute or substance implying another attribute or substance. It implies another term related to it. Both in relation to each other are called correlatives. For example, 'father' and 'child,' 'husband' and 'wife,' 'greater' and 'less,' 'cause' and 'effect,' 'murderer' and 'murdered,' are all pairs of correlatives, one member of a pair implying the other member. An *absolute* term is, on the other hand, a name of a substance or attribute, which does not imply another substance or attribute, as 'water,' 'air,' 'horse,' 'tree,' 'the solar system,' 'gold,' 'silver,' 'bird,' 'flower,' 'body,' 'man.'

§ 7. The next and last division of terms is into connotative and non-connotative. "A *connotative* term is one which denotes a subject and implies an attribute. By a subject is here meant

anything which possesses attributes[1]." A subject may be a substance, a phenomenon, or an attribute possessing another attribute. A connotative term has, in fact, two significations or meanings, one direct as applied to subjects, that is, to things or objects of thought possessing attributes, and the other indirect as implying attributes. For example, the term 'man' is connotative, inasmuch as it signifies directly each of an indefinite number of things or substances called men, and connotes or implies, at the same time, an attribute or collection of attributes, which is possessed, in common, by all men, and in virtue of which it is applied to them; the term 'metal' signifies likewise a number of substances taken separately, and implies, at the same time, the attribute or attributes which are common to them, and which distinguish them from other substances; the term 'colour' is connotative in as much as it stands for each of a number of attributes such as redness, blueness, greenness, &c., and connotes or implies, at the same time, an attribute in which those attributes agree. Similarly, the terms 'animal,' 'horse,' 'plant,' 'tree,' 'flower,' 'mineral,' 'house,' 'table,' 'paper,' 'figure,' 'virtue,' 'quality,' are all connotative, having each two significations, one direct, called the *denotation*, and the other indirect, called the *connotation* of the term. A *non-connotative* term is, on the other hand, "one which signifies a subject only or an attribute only," that is, it has only one signification, either of a thing, or of an attribute, and does not imply anything else. For example, the terms 'squareness,' 'visibleness,' &c., signifying each an attribute only, are non-connotative.

To the class of connotative terms belong the following:—(1) All concrete terms that are also general, or all general terms that are also concrete; for example, 'man,' 'bird,' 'fish,' 'river,' 'lake,' 'library,' 'nation' signifying directly an indefinite number of things, and implying attributes which they possess in common, are connotative[2]. (2) All abstract terms that are general,

[1] Mill's *Logic*, Vol. I. p. 31.

[2] To this head belong also adjectives when used substantively, that

or all general terms that are abstract. "Even abstract names," says Mill, "though the names only of attributes, may in some instances be justly considered as connotative; for attributes themselves may have attributes ascribed to them; and a word which denotes attributes may connote an attribute of those attributes[1]." As an example, he gives the term 'fault,' which denotes or signifies directly a quality, and connotes or signifies indirectly another quality, namely 'hurtfulness,' as an attribute of that quality. The general abstract terms 'virtue,' 'beauty,' 'quantity,' 'quality,' 'relation,' 'modality,' 'figure,' 'colour,' &c., are connotative[2]. Each of these terms denotes a number of attributes and connotes the attribute in which they all agree. 'Virtue,' for example, denotes justice, veracity, temperance, &c., and connotes the attribute in which they agree. 'Relation' denotes various kinds of relation, likeness or unlikeness, succession or coexistence, dependence or reciprocity, equality or inequality, and connotes the attribute in which they agree. Thus all general terms, whether concrete or abstract, are connotative. Whenever a term is general, that is, a name which is applicable to each of a number of objects of thought, whether the objects of thought be substances, phenomena, or attributes, it is connotative—denoting the objects of thought of each of which it is a name, and connoting the attribute in which the different objects of thought agree. A term cannot, in the same sense, be applied to each of a number of objects of thought, unless these objects of thought resemble each other in some attribute. The various objects of thought will be the denotation, and the common attribute the connotation, of the term. (3) Certain

is, as concrete general names or names of things implying an attribute or attributes.

[1] *Logic*, Vol. I. p. 33.

[2] To this head belong also adjectives when used as abstract general names, that is, as names of attributes, implying other attributes. For example, the adjective 'great' may denote an attribute as well as a thing, and connote the attribute 'greatness

singular terms which denote things, and connote or imply attributes belonging to those things, or convey some information about them. For example, the singular terms, 'the sun,' 'the first Emperor of Rome,' 'the only son of John Stiles,' 'the father of Socrates,' 'the author of the Iliad,' 'the present Prime Minister of England,' 'the present Viceroy of India,' &c. are connotative, inasmuch as they denote individuals, and connote or imply certain attributes belonging to them, or convey some information about them. To this head belong also the collective terms that are singular, such as 'the 76th Regiment of Foot in the British Army,' 'the University College Library,' 'the English people,' &c.

To the class of non-connotative terms belong the following:—(1) All singular abstract terms or terms signifying definite individual attributes, such as 'milkwhiteness,' 'equality,' 'squareness,' 'visibleness,' 'the figure of the desk before me,' 'the smell of the rose near me,' 'the colour of this piece of chalk,' &c. (2) Those singular terms (if there be any) which denote individual things or substances only, and do not connote or imply any attributes belonging to them. According to Mill all proper names belong to this class. "Proper names," says Mill, "are not connotative; they denote the individuals who are called by them; but they do not indicate or imply any attribute as belonging to those individuals. When we name a child by the name Paul, or a dog by the name Cæsar, these names are simply marks used to enable those individuals to be made subjects of discourse. Whenever the names given to objects convey any information, that is, whenever they have properly any meaning, the meaning resides not in what they denote but in what they connote. The only names which connote nothing are proper names; and these have, strictly speaking, no signification. A proper name is but an unmeaning mark which we connect in our minds with the idea of the object, in order that whenever the mark meets our eyes or occurs to our thoughts, we may think of that individual object. When we predicate (or affirm) of any thing its proper name; when we say, point-

ing to a man, this is Brown or Smith, or pointing to a city, that it is York, we do not, merely by so doing, convey to the reader any information about them except that those are their names[1]."

This view of proper names is contended against by Professor Jevons. "The connotation of a name," says he, "is confused with the etymological meaning or the circumstances, which caused it to be affixed to a thing. Surely, no one who uses the name England and knows what it denotes, can be ignorant of the peculiar qualities and circumstances of the country, and these form the connotation of the term[2].' Thus, according to Professor Jevons, all proper names, such as John Smith, Dartmouth, De Morgan, France, Socrates, Plato, &c., are connotative, signifying directly things, and implying the attributes or qualities belonging to them and distinguishing them from other individuals.

Neither Professor Jevons nor Mill stands alone in his view of proper names. Each has predecessors and followers in the same view; and the student ought to note the difference of opinion among logicians in regard to the true meaning of proper names. According to one school, they are non-connotative, being merely meaningless marks put upon individual things, while according to the other, they are connotative, denoting individuals and connoting qualities belonging to those individuals. The question is a philological and a psychological one, and cannot be discussed here. Mill's view is true if a proper name always means what it does, when it is first used as a symbol or sign for an individual thing. At that stage no attribute is associated with the name. But as our knowledge of the individual thing increases, we associate its attributes with the name, which suggests afterwards not only the individual thing, but also the attributes. A proper name would, therefore, appear to be at first without any connotation or signification of attributes, but it seems to acquire this signification as our knowledge of the individual becomes

[1] Mill's *Logic*, Vol. I. pp. 36—37. [2] Jevons's *Lessons*, pp. 42—43.

more and more definite, as its name becomes associated in our mind with its attributes, and as the attributes become a means of distinguishing that individual from others belonging to the same class or species.

If a term has more than one connotation, it is ambiguous. It is then really equivalent to two or more terms, and should be treated as such; the terms 'thing,' 'substance,' 'right,' 'thought,' 'foot,' 'church,' 'faith,' 'feeling,' 'form,' 'government,' &c., having more than one signification, are ambiguous, and each of them is really equivalent to more than one term. For example, the term 'thing' means popularly a body, something tangible and visible, or an object of sense; but, in a wider sense, it means also a mind, any substance, mental or material, as when we speak of mind as a 'thinking thing'; and, in a still wider sense, it means attributes and phenomena as well as substances, as when sensations, ideas, feelings, hopes, joys, sorrows are spoken of as 'mental things.' The term 'substance' is also ambiguous, signifying popularly a thing consisting of attributes, and philosophically a substratum or basis in which all the attributes of a thing are inherent, or which forms an inexplicable tie or bond among them.

§ 8. All the above described divisions of terms are based on the following general aspects of things. The fact that there are individual things, gives rise to the Singular Term. The fact that the same attribute or collection of attributes is found in many individual things, gives rise to the General Term. The fact that many individual things may sometimes be taken together and regarded as constituting a whole, gives rise to the Collective Term. The fact that one attribute may be spoken about and treated of apart from others with which it exists in an individual thing, gives rise to the Abstract Term; and the fact that it really exists in combination with others in an individual thing or substance, and cannot exist by itself, gives rise to the Concrete Term. The fact that certain attributes and things are necessarily connected with one another, and imply each other, gives rise to Correlative Terms; and the fact that

others are not so connected and do not imply each other, gives rise to the Absolute Term. The fact that our knowledge of things is progressive, that we first come to know one attribute of a thing or of a group of things and then another, gives rise to the Connotative Term; or rather the fact that the name given to a thing or a group of things comes with the progress of our knowledge of the thing or things, to be associated with this additional knowledge, and becomes afterwards a sign for it, gives rise to the distinction of Connotative and Non-connotative Names. The Negative Term shows that things may be named not only by the attributes which they actually possess (as in the case of Positive Terms), but also by those which are absent in them; that names may be applied to things in virtue of the absence of some as well as of the presence of other attributes; that things may be distinguished into classes by their negative as well as by their positive qualities.

§ 9. Exercises.

In describing the logical characters of a term, the following method should be followed:—

I. What is given is a word or combination of words. Ascertain its meaning, and see whether it is capable of being employed by itself as the subject or the predicate of a proposition. If it is not, then it is syncategorematic; if it is, then it is categorematic, that is, a term.

II. In the latter case, proceed to describe the logical characters of the term in the following order[1]:—

i. Whether it is single-worded or many-worded.

ii. Whether it is singular or general.

iii. Whether it is collective and singular, or collective and general.

iv. Whether it is concrete or abstract.

[1] I have not given here the distinction of categorematic or syncategorematic as a logical character of terms, as it is applicable to words rather than to terms. Single words and combinations of words should be distinguished into categorematic and syncategorematic, and not terms.

v. Whether it is positive, negative, or privative.
vi. Whether it is absolute or correlative.
vii. Whether it is connotative or non-connotative.

III. If it has more than one meaning, then describe its logical characters, first in accordance with the most obvious or usual meaning, and then in accordance with the other meaning or meanings in order of importance.

Examples.

1. 'Man':—categorematic; single-worded; general; concrete; positive; absolute; connotative.

2. 'Mankind':—categorematic; single-worded; collective and singular; concrete; positive; absolute; connotative.

3. 'The Sun':—categorematic; many-worded; singular; concrete; connotative; positive; absolute.

4. 'Beautiful':—categorematic (according to some syncategorematic; because the complete term consists of the word 'beautiful' and a word understood after it, such as 'thing,' or 'person,' &c., for example 'that picture is beautiful': here the complete sentence is that 'that picture is a beautiful thing'); single-worded; general; concrete; positive; absolute (correlative, if 'beautiful' is regarded as implying 'ugly'); connotative.

5. 'Equal':—its logical characters are the same as those of 'beautiful,' except that it is correlative, *i.e.*, it implies something that is equal to it. 'Larger,' 'greater,' 'upper,' &c., are also correlative.

6. 'Lame,' 'dumb,' 'blind,' have the same logical characters as 'beautiful,' except that they are *privative.*

7. 'Army':—categorematic; single-worded; collective, when it means some one army, *i.e.*, in the sense of 'an army,' but general when it means different armies, and connotes the attributes possessed in common by them; concrete; positive; absolute; connotative.

8. 'Rational animal,' 'flowering plant,' 'metal conducting heat and electricity,' 'animal living in water':—categorematic; many-worded; general; concrete; positive; absolute; connotative.

9. 'The figure of this body,' 'the luminosity of this flame, 'the smell of this rose':—categorematic; many-worded; singular; abstract; positive; absolute; non-connotative.

10. 'Quantity':—categorematic; single-worded; general; abstract; positive; connotative.

11. 'Humanity':—categorematic; single-worded; abstract; positive; absolute; general and connotative, if 'humanity' admits of any variety or division; singular and non-connotative, if 'humanity' is something individual, that is, incapable of any variety or division.

N.B. Sometimes it is very difficult to describe the logical characters of a term,—the difficulty arising chiefly from difference of opinion as to the real nature of the thing signified by the term,—as to the real meaning or meanings of the term, &c. Take, for example, the term 'phenomenon.' It is general; connotative; concrete; positive; but is it absolute or correlative? According to some philosophers, it implies the existence of 'nöumenon,' and is, therefore, correlative, while according to others who do not believe in the existence of nöumena, it is *absolute*. Similarly, the term 'attribute' is either relative to 'substance' or absolute according as the existence of the latter is believed in or not. 'Cause' is evidently related to 'effect,' and 'effect' to 'cause.' 'Antecedent' to 'consequent,' and the latter to the former. Are 'time' and 'space' abstract or concrete, singular or general, absolute or correlative? The answer to this question will be given differently by different philosophers.

Examples for Solution.

Describe the logical characters of the following:—

I. (1) Man, (2), good man, (3) human, (4) humanity, (5) humanitarian, (6) humanitarianism, (7) A man whom I saw yesterday.

II. (1) Five, (2) fifth, (3) five attributes, (4) five bodies, (5) these five metals.

III. (1) Good, (2) the good, (3) goodness, (4) goods, (5) the highest good, (6) a good quality, (7) great goodness.

IV. (1) Book, (2) library, (3) a library, (4) Encyclopædia, (5) Encyclopædia Britannica.

V. (1) Organ, (2) organic, (3) inorganic, (4) organism, (5) an organism, (6) organic being.

VI. (1) Nation, (2) a nation, (3) national, (4) nationality, (5) nationalities.

VII. (1) Strong, (2) strength, (3) the strong, (4) strong man, (5) strength of character, (6) this strong man.

VIII. (1) Element, (2) elementary, (3) elementary attribute, (4) elementary substance, (5) the 'Elements of Euclid,' (6) a chemical element.

IX. (1) Plant, (2) figure, (3) inconvenient, (4) blindness, (5) business, (6) universe, (7) heat.

X. (1) Multitude, (2) the first emperor, (3) irreligious, (4) virtue, (5) mind, (6) matter, (7) body, (8) form.

XI. (1) Atmospheric air, (2) organization, (3) life, (4) force, (5) time, (6) space, (7) cause, (8) motion, (9) substance, (10) being, (11) something, (12) nothing.

XII. (1) Sense, (2) rest, (3) speed, (4) law, (5) the circle of sciences, (6) gravity, (7) spirit, (8) higher, (9) right, (10) sensation, (11) knowledge, (12) feeling, (13) perception, (14) smell, (15) vision, (16) taste, (17) colour, (18) relative.

XIII. (1) His Majesty, (2) His Honour, (3) Her Serene Highness, (4) elementary atoms, (5) the passage of water to the state of ice, (6) soluble in water, (7) the surfaces of bodies, (8) the number of the metals, (9) the gaseous envelope encircling the earth, (10) the theory of ideas, (11) the undulatory theory of light, (12) to reason against any of these kinds of evidence, (13) the yellowness of gold, (14) the lightest substance known, (15) the perception of the external world, (16) consciousness.

XIV. (1) "The place which the wisdom or policy of antiquity had destined for the residence of the Abyssinian princes."

(2) To attend accurately to the operation of our minds.

(3) The ignition of phosphorus.

(4) A just interpretation of nature.

(5) A series of electric discharges.

XV. (1) Co-existence, (2) succession, (3) identity, (4) resemblance, (5) causation, (6) equality, (7) relation, (8) subsistence.

CHAPTER II.

The Denotation and Connotation, Division and Definition, of Terms.

§ 1. In the preceding chapter, we have seen that most terms denote or signify directly things, and connote or imply attributes belonging to them, that is, have, at the same time, two meanings, of which one is called their denotation, and the other their connotation. The denotation of a term consists of the individual things to each of which the term is, in the same sense, applicable. The connotation of a term consists of the attribute or collection of attributes implied by the term, and possessed by each of the individual things denoted by it. For example, the denotation of the term 'man' consists of all the individual things, called 'men,' whether now living or dead,—of all things, in fact, to which the term 'man' is applicable; while its connotation consists of the attributes, say 'animality' and 'rationality,' implied by it, and possessed in common by all men. The denotation of the term 'book' consists of all the various kinds of books written in all languages throughout the world, while its connotation consists of the attribute or attributes which all books possess in common, and which are implied by the term 'book.' The term 'triangle' in denotation signifies all the different kinds of triangles,—the individual things called triangles, while in connotation it signifies the attribute possessed in common by all triangles, namely, the attribute of being bounded by three lines.

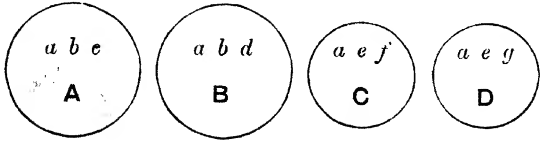
When a term signifies an individual, *i. e.*, has for its denotation only a single object or thing, its connotation is the group of attributes possessed by the individual thing, and signified by the term. For example, the term 'the sun' has for its denotation one individual thing only, while its connotation consists of the attributes possessed by that individual thing, and implied by the term; the term 'the present Prime Minister of England' denotes an individual person, and connotes 'the attribute of being the Prime Minister of England'; the term 'the father of Socrates' denotes a person, and implies 'the attribute of being Socrates's father'; thus all singular terms have both a denotation and a connotation, proper names alone, according to Mill, being excepted. We have already alluded to the difference of view among logicians on this point, and need not here revert to it.

§ 2. The denotation and the connotation of a term have a close relation to each other. When the denotation of a term is increased or decreased, its connotation is decreased or increased; again, when the connotation of a term is increased or decreased, its denotation is decreased or increased. If you add a new group of things to the group denoted by a term, you subtract one or more attributes from its connotation. Include a new class within a class signified by a term, and its connotation will lose a part of its meaning, that is, the attributes possessed in common by all the individuals of the enlarged class will be fewer in number than before. The term 'man' has for its denotation the group of animals called men, and for its connotation the two attributes, 'animality' and 'rationality.' If its denotation is enlarged by including in it 'irrational animals' or all other animals than man, its connotation will no longer be the same as before, but consist of that attribute only which is possessed by all the members of the newly formed enlarged class, namely, the attribute 'animality,' and thus lose the other attribute 'rationality.' The term 'triangle' will likewise lose an attribute—'three sidedness'—from its connotation, when new groups or classes, such as 'quadrilaterals' and 'multilaterals,' are added to its denotation. The term 'animal' will lose such attributes as sensibility, loco-

motion, &c., from its connotation, when its denotation is enlarged so as to include 'plants' in its sphere, the new denotation and connotation giving rise to the new term 'organized being.' This term will again lose a part of its connotation, when its denotation is enlarged by the addition of 'inorganic things,' the increased denotation and the decreased connotation giving rise to the term 'material being' or 'body,' including inorganic as well as organic beings. Thus, we see that addition to the denotation of a term implies subtraction from its connotation, and that the new class thus produced is generally signified by a new term with a smaller connotation. Similarly, it can be shown that, when the denotation of a term is decreased, its connotation is increased. Again, if you add a new attribute to the attribute connoted by a term, you subtract a group of things from its denotation. The examples we have just given illustrate this. Add the attribute 'organization' to the connotation of the term 'material body,' the attribute 'sensibility' to the connotation of the term 'organized being,' the attribute 'rationality' to the connotation of the term 'animal,' the attribute 'three-sidedness' to the connotation of the term 'rectilineal figure'; and, in each case, the denotation of the corresponding term is decreased, that is, a smaller number of things possess the added attributes; and the increased connotation and the decreased denotation give rise to a new term. Similarly, it can be shown that, when the connotation of a term is decreased, its denotation is increased.

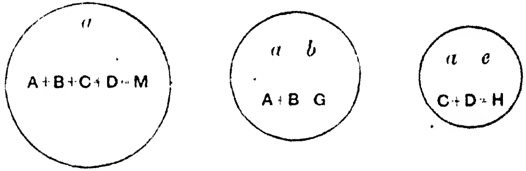
§ 3. The relation between the denotation and the connotation of a term may be explained by figures as follows:—

Let A, B, C, D, be four general terms, their denotations being represented by the circles A, B, C, D, and their connotations



by the small letters in them. First, if the things represented by A, and those by B, be brought under one class, the connotation of the name of this class will be the two attributes, *a* and *b*, common to A and B. Again, if C and D be brought together under a new class, the connotation of the name of this class will be the two attributes, *a* and *e*, common to C and D. Similarly, if the denotation of the two new classes be brought together to form a still larger class, the connotation of the name of this class will be still smaller, the attribute *a* being the only one common to all the circles.

Secondly, if the connotation of the largest class M, be increased by *b*, then its denotation will be only A + B, or



M − (C + D); and if by *e*, then its denotation will be C + D, or M − (A + B); that is, addition to the connotation of a term causes subtraction from its denotation. Again, if the connotation of A + B be increased by *c*, then its denotation will be only A, or A + B − B; if by *d*, then only B, and so forth.

It should be observed that the denotation of a term will neither increase nor decrease, if its connotation increases by any attributes that are found to be possessed by all the members of the class, or that follow from any part of the connotation. Thus, there will be no alteration in the size of the circle A, if the attributes *m* and *n* be added to *a*, *b*, *c*, provided that *m* and *n* are found to be possessed by all A, or follow from *a*, *b*, *c*. Similarly, the connotation will remain unaltered, if the circle is enlarged by the addition of any individuals that are found to possess the known marks or attributes of the class. Thus *a*, *b*, *c*

will neither increase nor decrease, if the circle A increases by the birth, manufacture, or discovery of new individuals possessing the attributes *a*, *b*, *c* of the class.

The denotation and the connotation of a term are not absolutely fixed. Both may increase or decrease with the advance of knowledge. Given the connotation of a term, its denotation is more or less indefinite. Given the denotation, the connotation is more or less indefinite. Suppose, for example, that the term 'metal' has for its connotation the three attributes *a*, *b*, *c*, what is then its denotation? Every individual thing that possesses those three attributes. Not only the metals at present known but all substances that may hereafter be found to possess those three attributes, will be included in its denotation; thus the circle representing the denotation of the term 'metal' may go on increasing with the progress of discovery in chemistry. Or some substances that are now recognized as metals may turn out to be compound; and thus the circle may decrease in extent with the progress of chemical analysis. Suppose, on the other hand, that the denotation of the term 'metal' is fixed and definite, that is, consists of a certain number of known elements, and is represented by a certain circle, what is then its connotation? The attributes connotated by the term 'metal,' and possessed in common by all the substances denoted by it. Now, these attributes may increase in number with the progress of chemical knowledge, and the term 'metal' may afterwards come to connote many attributes which it does not at present. Thus, both the denotation and the connotation of a term may vary with the increase of knowledge.

§ 4. Exercises:—

1. Describe the change in the denotation and connotation of each of the terms in the following series as you pass from the 1st to the 2nd, from the 2nd to the 3rd, and so forth, and, again, in the reverse order, as you pass from the last to the last but one, and so on.

i. Element, metal, gold.

ii. Animal, man, Englishman.

iii. Right-angled triangle, triangle, rectilineal figure, figure.

iv. Literature, English literature, philosophical literature in English.
v. Force, gravity, the mutual attraction of the sun and the earth.
vi. Solid, stone, precious stone, ruby.
vii. Rock, igneous rock, volcanic rock, pumice.

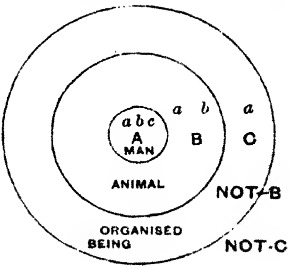
2. Give as many examples as you can of series of three, four, or more terms each, in which each term of greater extension stands before a term of less extension.

3. "The denotation and the connotation of a term vary *inversely*." Explain and criticise this statement.

4. Can you give any example of terms whose denotation may increase without any change in the connotation, and also of terms whose connotation may increase without any change in the denotation?

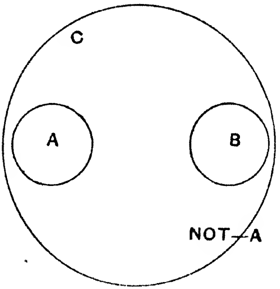
5. What determines the denotation and the connotation of a term? Has every term a denotation and a connotation?

§ 5. If a number of terms be related to one another as represented in this figure,—that is, if the denotation of A be contained in that of B, and if the denotation of B be contained in that of C, and if their connotations be as shown in the figure by the small letters, then C is called a genus in relation to B, and B a species in relation to C; B a genus in relation to A, and A a species in relation to B: that is, the containing and the contained term are called respectively genus and species in relation to each other. The distinction between them is however relative, for the same term may be a genus in relation to one, and a species in relation to another; here, for instance, B is a genus in relation to A, and a species in relation



to C. The attribute 'b' is called the differentia of the species B in relation to the genus C, and the attribute 'c' the differentia of the species A in relation to the genus B. The differentia of a species is that attribute which being added to the connotation of the genus gives the connotation of the species; here the attribute 'b' being added to 'a' the connotation of the genus C, gives *ab*, the connotation of the species B, and is thus the differentia of the species B. By the differentia a species is distinguished from the other species contained in the same genus; C as a genus, for example, contains two species B and not-B, that is, those C's that are B, and those C's that are not-B; and by the differentia 'b' the species B is distinguished from the other species not-B contained in the same genus C. The two species B and not-B included in the genus C are called co-ordinate species. In the figure on page 54, the three sub-classes A, B and C contained in the class G are, similarly, co-ordinate species of the genus G; and the terms A, B, and C are called *co-ordinate* in relation to each other and *subordinate* in relation to G, while G is called *super-ordinate* in relation to them. C and not-C are called *contradictory* terms or concepts, not-C including everything except C: that is, C and not-C cover the whole sphere of thought and existence; every thing and every thought is included in either C or not-C. A and not-A, B and not-B, taking not-A and not-B in their widest sense, are also contradictory terms, and cover the whole sphere of thought and existence. Two contradictory terms are so related to each other, that both can be neither affirmed nor denied of one and the same thing, that if one be true, the other must be false, and if one be false, the other must be true, of one and the same thing. For example, both the terms 'organized being' and 'not-organized being' cannot be affirmed of one and the same thing, nor can both be denied of it; if 'organized being' be affirmed, 'not-organized being' must be denied, and if the latter be affirmed, the former must be denied, of a thing; for every possible thing must fall into one or other of the two comprehensive classes which divide between them the whole

sphere of thought and existence; a thing not included in one or other of the two all-embracing classes, has existence neither in nature nor in thought. But if two terms be so related to each other, that both cannot be affirmed, but that both may be denied, of one and the same thing, that if one be true, the other must be false, but, not conversely, if one be false, the other must be true, of it, then they are called *contrary terms*. For example, of the two terms 'black' and 'white,' if 'black' be affirmed, 'white' must be denied, of one and the same thing, but, not conversely, if 'black' be denied, 'white' must be affirmed, of it, for both may be denied of it, that is, the thing in question may be neither black nor white, but of some other colour or of no colour at all. Thus 'cold' and 'hot,' 'up' and 'down,' 'virtue' and 'vice,' 'light' and 'darkness,' &c., are contrary terms, while 'cold' and 'not-cold,' 'hot' and 'not-hot,' 'light' and 'not-light,' &c., are contradictory terms. Two contrary terms do not completely cover the whole sphere of thought and existence, while two contradictory terms do. The difference between them may be thus shown by diagrams:—Suppose that all the different kinds and shades of colour are represented by the large circle C, then the two contrary terms 'black' and 'white' are represented by the two small circles, A and B, lying outside each other, but both falling under the circle of colour C, while the two contradictory terms 'black' and 'not-black' are represented, respectively, by A and not-A, which jointly cover the whole sphere of thought and existence, not-A including everything except A.



Exercises on the Mutual Relations of Terms.

1. Give the genus, species, and differentia of the following terms :—

   (1) Plant, (2) Figure, (3) Triangle, (4) Body, (5) Metal, (6) Element, (7) Book, (8) Flower, (9) Rock, (10) Mind.

2. Give a subordinate, a super-ordinate, and a co-ordinate of the following terms:—

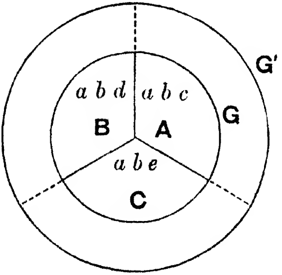
   (1) Animal, (2) Solid, (3) Virtue, (4) Rock, (5) Substance.

3. Give the contradictory and a contrary of the following terms :—

   (1) White, (2) Simple, (3) High, (4) Liquid, (5) Good, (6) Moral, (7) Vice, (8) Mortal, (9) Animal, (10) Mind, (11) Matter, (12) Form, (13) Beautiful.

4. Has every term a genus and species?

§ 6. Division and Definition of Terms :—The orderly statement of the denotation of a term, or the grouping of the denotation into smaller classes according to the presence or absence, or varying degree of an attribute, is the division of the term. And the setting forth of the connotation of a term is its definition, or the definition of the things or class denoted by the term. The definition is more or less complete according as the connotation of a term, or the group of attributes in which the things agree, is more or less exhaustive. The definition of a



term, being a statement of its connotation, varies with any change in the latter. The division of a term likewise varies with its denotation. With the increase in denotation the sub-classes increase in number or in extent. If A, B, C are smaller classes under G, and if G is enlarged into G', A, B, C will no longer cover the whole extent. They must increase in extent as represented by the dotted lines, or the larger class

must be divided in a different way, and give rise to new sub-classes.

The concept, like the term, has its content or comprehension, and its extent or extension. The extent of a concept consists of the individual conceptions or things in which its content is found. The content of a concept consists of the elementary notions or ideas which constitute its very essence and meaning. The statement in words of all or any of these elements, is the definition of the concept; and the grouping of the individual conceptions into minor divisions according to their resemblance and difference, is the division of the concept. The extent and the content of a concept, and the relation between them, may be represented by circles, and capital and small letters of the alphabet, just as in the case of a term.

§ 7. Definition as a logical process is the process of determining the connotation of a term, or the attributes possessed in common by the things denoted by the term. It implies observation, analysis, abstraction, comparison, and even generalization, and is a most important process in science. A definition as a product of thought is the product of this process. In a complete treatise on Logic, Definition would deserve a most prominent place. Here I shall give only the rules to which a definition ought to conform, noting, by the way, the faults to which the violation of them gives rise. A definition should conform to the following rules or conditions:—

(1) That it be an analytical statement of the connotation of the term defined. This rule includes the one given by the older logicians, that a definition should be *per genus et differentiam*, that is, a statement of the genus and a differentia of the term. If a part of the connotation is stated, the definition is *partial* or *incomplete;* and if the whole of it is stated, the definition is *complete*. An incomplete definition, if it serves to distinguish the things denoted by the term from others belonging to the same higher class, corresponds to a definition *per genus et differentiam*, while a complete definition corresponds to a definition *per genus et differentias.* The violation of this rule gives rise to

what has been called an accidental definition, or a mere description of the things denoted by the term as well as to redundant and incomplete definitions. When any attribute not possessed by all the things denoted by the term, or not forming a part of its connotation, is stated in the definition, it is *accidental;* and when some attributes that follow from its connotation are stated, it is *redundant.* For example, 'a triangle is a figure which is bounded by three straight lines, and which has all its angles together equal to two right angles' is a redundant definition; 'water is a liquid substance' is incomplete; 'man is a cooking animal' and 'iron is the cheapest metal' are accidental; and 'a plant is an organism having roots, branches, leaves, flowers, fruits, &c.,' is a mere description.

(2) That it exactly coincide in extent with the denotation of the term defined. In other words, it should not include things other than those to be defined, nor should it exclude any of them. The violation of this rule gives rise to the fault of too great width or narrowness. For example, the definitions 'man is a sentient being,' 'a metal is a solid substance,' are too wide; while 'man is a civilized animal,' 'a metal is a heavy element,' are too narrow.

(3) That it do not contain the term to be defined, or any of its synonyms. The violation of this rule gives rise to the fault of the circle in definition. For example, when a term is defined by itself, as 'man is a *human* being,' 'a plant is a *vegetable* organism,' 'life is the sum of the *vital* functions,' or when a term is defined by a second term, and the second again by the first, as 'man is a rational animal'; and, again, 'a rational animal is a human being,' 'matter is an extended substance'; and, again, 'an extended substance is a material body.' From this rule it is evident that a term connoting an elementary attribute cannot be defined. For its definition will contain either the term itself or its synonym, or be merely a description of it. Hence such terms as 'consciousness,' 'feeling,' 'pleasure,' 'pain,' 'colour,' 'smell,' &c., connoting elementary attributes, cannot be defined. The definitions or rather descriptions and analyses that

are given of them, consist of a statement of the circumstances or conditions under which they are produced.

(4) That it be expressed in clear and unambiguous language. In other words, it should not be expressed in obscure, figurative, or ambiguous language. The violation of this rule gives rise to obscure and figurative definitions, which are misunderstood, often bear more than one interpretation, and are a source of much trouble and misunderstanding both to the teacher and the pupil, as well as to the general reader. 'The intuitive reason is the eye of the soul,' 'The mind is a tabula rasa,' 'The ideas are the images of external objects,' 'The soul is the first *entelecheia* of a natural body which has potential life[1],' will serve as examples.

(5) That it be not negative, where it can be affirmative. The violation of this rule gives rise to negative definitions which are often almost meaningless. 'Mind is not matter,' 'mind is the non-extended,' 'evil is that which is not good,' 'vice is that which is not virtue,' 'virtue is that which is not vice,' are negative definitions that are almost useless.

Exercises on Definition.

I. Test the following definitions:—

(1) Logic is the science of thought.

(2) Logic is the science of reasoning.

(3) A triangle is a three-sided figure, of which any two sides together are greater than the third.

(4) (*a*) "A power is a force which tends to produce motion,"—Ganot.

(*b*) "Porosity is the property which bodies possess of having pores."—Ganot.

(*c*) "Matter is any thing whose existence can be determined by one or more of our senses."—Ganot.

(5) A force is a power that can produce motion.

(6) A plant is an insentient organized being.

(7) A crystal is a solid substance of a definite geometrical form.

[1] Aristotle's definition of the soul quoted by Reid in his *Inquiry*.

(8) An equilateral triangle is a three-sided figure, having all its angles and sides respectively equal to each other.
(9) A triangle is a figure bounded by three straight lines.
(10) Logic is the science of human knowledge.
(11) Gold is a precious metal.
(12) Diamond is a kind of carbon.
(13) Oxygen is a supporter of combustion.
(14) A rock is a hard substance.
(15) Inorganic substances are dead material bodies.
(16) Mind is a thinking substance.
(17) A plant is a being possessing vegetable life.
(18) A glacier is a river of ice.

II. Define the following terms:—

(1) Student, (2) College, (3) University, (4) Library, (5) Class, (6) Term, (7) Mind, (8) Matter, (9) Thing, (10) Food, (11) Bird, (12) Lake, (13) Book, (14) Tree, (15) Plant, (16) Flower, (17) Animal, (18) Virtue, (19) Religion, (20) Science.

§ 8. Logical division is to be distinguished, on the one hand, from what is called physical division, or the analysis or separation of an individual thing into its component parts; and, on the other, from what is called metaphysical division, or the analysis of an individual thing into its constituent attributes, qualities, or properties.

The division of a plant into its roots, trunk, branches, and leaves, or of an animal into its head, trunk, limbs, &c., is physical; while the division into the qualities which constitute a plant or an animal is metaphysical. The division of a piece of gold into two or more parts is physical, while the division or rather the analysis of it into the qualities, yellow colour, a certain specific gravity, a certain form, size, solidity, &c., which are possessed by every particle of it, is metaphysical. Similarly, every individual object may be divided physically into its component particles or parts, and metaphysically into its qualities, properties, or attributes. But both these kinds of division should be distinguished from logical division, which cannot be

applied to an individual thing or attribute, but only to a class of things or attributes.

The rules or conditions to which a logical division ought to conform are the following :—

(1) That what is to be divided be a class and not an individual. In other words, a singular term cannot be divided, and only a general term is capable of logical division. The violation of this rule gives rise either to physical partition, or to metaphysical analysis. A collective term, such as 'a nation,' 'a library,' 'a forest,' 'the universe,' 'the animal kingdom,' being really singular in signification, is also incapable of logical division.

(2) That the division be founded upon the presence or absence, or upon the varying degree, of a certain *fundamental* attribute; in other words, that there be only one *fundamentum divisionis* or principle of division. The violation of this rule gives rise to the fault of cross-division.

(3) That the name of the class divided be applicable, in the same sense, to each of the sub-divisions or smaller classes into which the whole is divided. The violation of this rule also gives rise to physical partition, or to metaphysical analysis.

(4) That the sub-divisions be together equal to the class divided. In other words, the denotations of the dividing terms should together exactly coincide with the denotation of the divided term. The violation of this rule gives rise to the fault of incomplete or over-complete (too narrow or too wide) division.

(5) That the sub-divisions do not overlap, but completely exclude each other. In other words, any individual included in the denotation of one dividing term, should not be included in the denotation of another. The violation of this rule gives rise to the fault of over-lapping division.

I shall illustrate the above rules by a few examples :—(1) A division of rectilineal triangles is into (i) equilateral, (ii) isosceles, and (iii) scalene. Here the term divided is general; the principle of division is the equality or inequality of the sides; the divided term 'rectilineal triangle' is applicable to each sub-

division; the sub-divisions taken together coincide exactly with the class divided; and they exclude each other. In this division an isosceles triangle is defined as having *only two* sides equal, otherwise the second sub-division will include the first, and the division involve the fault of overlapping. (2) A division of rectilineal figures is into (i) three-sided, (ii) four-sided, (iii) five-sided, (iv) six-sided, (v) more-than-six-sided; here the divided term is general; the principle of division is the varying number of the sides; the term 'rectilineal figure' is applicable to each sub-division; all the sub-divisions are together equal to the whole class; and they exclude each other. (3) A division of plane angles is into (i) acute, (ii) right, and (iii) obtuse; this also conforms to the five rules.

From the examples given above it is evident that we cannot, without a knowledge of the things divided, ascertain whether a division conforms to the rules. There is, however, one kind of logical division in which this is evident from the form. It is called Dichotomy—the dividing or cutting into two. In this kind of division a class is divided into two parts, which, according to the Principle of Excluded Middle, completely cover the whole. Its nature will be evident from the following examples:—

(1) ANIMALS

- Vertebrate animals
  - Mammalia
  - Vertebrate animals other than Mammalia
    - Birds
    - Other than birds
      - Reptilia
      - Other than Reptilia
        - Amphibia
        - Other than Amphibia (Fishes)
- Invertebrate animals

(2) MATERIAL BODIES

- Solid bodies
- Not-solid bodies
  - Liquids
  - Not-liquid bodies
    - Gaseous
    - Not-gaseous (*i.e.* other than solid, liquid and gaseous bodies)

(3) EXISTENCES, OR, THINGS IN THE WIDEST SENSE

- Material
  - Organized (plants and (animals)
    - Sentient (animals)
      - Rational (man)
      - Irrational (lower animals)
    - Insentient (plants)
      - Flowering
      - Flowerless
  - Not-organized (inorganic things, rocks, minerals, &c.)
- Immaterial (mind, soul, spirit, &c.)

In these examples of division by Dichotomy, the rules given above hold good. In Deductive Logic, we can, strictly speaking, treat only of this kind of Logical Division. For, in no other kind of it, can we feel perfectly sure, without special reference to the things divided, that the rules hold good: that the sub-groups taken together, for example, are neither greater nor less than the whole divided; that they do not overlap; or that there are not more principles of division than one. The reader can easily satisfy himself of the truth of this remark, by trying to find out for himself whether the following divisions are strictly logical or not:—

1. The Division of Invertebrate animals into (1) Protozoa, (2) Cœlenterata, (3) Annuloida, (4) Annulosa, (5) Mollusca.

2. The Division of Mental Phenomena into (1) Cognition, (2) Feeling, (3) Volition.

3. The Division of Plants into (1) Monocotyledons, (2) Dicotyledons, and (3) Cryptogams.

4. The Division of Rocks into (1) Igneous, (2) Aqueous, and (3) Metamorphic.

Exercises on Division.

I. Test the following Divisions:—

1. Triangles into Equilateral, Right-angled, and Scalene.
2. Terms into Abstract, Absolute, and General.
3. Terms into Singular, General, Collective, and Distributive.
4. Figures into Triangles, Quadrilaterals, and Circles.
5. Quadrilateral Figures into Parallelograms, Squares, Oblongs, Rhombuses, and Rhomboids.
6. Flowers into Petals, Sepals, Stamens, and Pistils.
7. The World into Asia, Africa, Europe, Australia, and America.
8. Deductive Logic into Terms, Propositions, and Inferences.
9. A piece of Chalk into Whiteness, Extension, Solidity, Weight.
10. The animal body into the Lungs, the Heart, the Stomach, the Senses, the Brain, the Muscles, the Bones, and the Ligaments.
11. Terms into Concrete, Singular, Positive, and Abstract.
12. Houses into Brick-made, Stone-made, One-storeyed, Two-storeyed, and Huts.
13. Religion into Christian, Mahomedan, Hindu, and Parsi.
14. Virtue into Truthfulness, Justice, Benevolence, Temperance.
15. Sciences into (1) Theoretical and Practical, (2) Material and Mental, (3) Mathematical, Physical, and Moral.
16. Substances into Material, Organic, Inorganic, and Mental.
17. Logic into Deductive, Inductive, Formal, and Material.
18. Things into Material, Immaterial, Sentient and Insentient.

II. Divide logically the following terms:—

(1) Name, (2) Proposition, (3) Book, (4) House, (5) Student, (6) Examination, (7) Act, (8) War, (9) Phenomenon, (10) Man, (11) Colour, (12) Smell, (13) Taste, (14) Touch, (15) Sound, (16) Force, (17) Energy, (18) Body, (19) Mental State, (20) Paper.

PART II.—PROPOSITIONS.

CHAPTER I.

The Definition and Divisions of Propositions.

§ 1. A proposition may be defined as an affirmation or denial of a certain relation between two terms. It thus consists of two terms and of a word, or words, or part of a word expressed or understood, as a sign of affirmation or denial. That which is affirmed or denied is called the Predicate, that of which it is affirmed or denied is called the Subject, and that which stands as a sign of affirmation or denial is called the Copula, of the proposition. For example, in the proposition "All men are mortal," 'all men' is the subject, 'mortal' the predicate, and 'are' the copula or the sign of affirmation; in the proposition "Some men are not wise," 'some men' is the subject, 'wise' the predicate, 'are not' the copula or the sign of denial; in the proposition "The sun rises," 'the sun' is the subject, 'rise' the predicate, and the letter 's' is the copula; here the affirmation of the predicate of the subject is expressed by a slight alteration, called an inflection of the word 'rise.' When fully expressed, the last proposition stands thus:—"The sun is rising," in which the sign of affirmation is explicitly stated, and is the same as in the first example given above.

The subject or the predicate of a proposition may consist of a single word or of any combination of words constituting a term.

In the propositions "Chalk is white," "The virtuous are happy," "That all men are mortal is known to everybody," "To know any subject thoroughly is not easy," &c., 'chalk,' 'the virtuous,' 'that all men are mortal,' 'to know any subject thoroughly' are, respectively, the subjects, and 'white,' 'happy,' 'known to everybody,' 'easy' are, respectively, the predicates.

The copula of a proposition, when stated in the logical form, consists usually of the parts of the verb 'to be' with or without the negative particle 'not.' It should be carefully noticed that the copula merely expresses a certain relation between the subject and the predicate, and does not imply the existence of either. For example, in the symbolical proposition 'A is B,' 'A' is the subject, 'B' the predicate, and 'is' the copula which, in the affirmative form, merely expresses the presence of a particular relation between A and B, and does not imply the existence of either the subject or the predicate. Similarly, in the proposition 'A is not B,' the copula 'is not' is merely a sign of the absence of a particular relation between A and B, and does not signify either the existence or the non-existence of A or B. The verb 'to be' used as copula should be distinguished from the same verb used as copula and predicate in a proposition. In the latter case, it implies the existence of the subject. In the proposition, 'A is,' for example, 'is' means 'exists' and is equivalent to 'is existing.' In this sense, also, the verb 'to be' is ambiguous; for the words 'is,' 'are,' 'being,' &c., like 'exists,' 'existing,' 'existence,' &c., may, according to context, mean either existing in Thought, that is, free from self-contradiction, or existing in Nature, that is, corresponding to actual existence, and free not only from self-contradiction but also from disagreement with fact or reality. The proposition, 'A is,' may mean simply that the idea or concept A exists in Thought without any reality or fact corresponding to it, or it may mean that the idea A exists in Thought and agrees with fact or reality. The subject of a proposition may exist in neither of these senses. In the proposition, "A square circle is not," the subject 'a square circle' has existence neither in Nature nor in Thought.

According to some logicians the copula consists of the verb 'to be' without the negative particle 'not.' They attach this particle not to the copula but to the predicate, and thus make all propositions apparently affirmative. They cannot of course get rid of the *not*, or of the fact of negation or absence of something; and what is excluded from the copula must be included in some form in the predicate. In the proposition 'A is not B,' 'not-B' is, according to them, the predicate, and 'is' the copula; the 'not' of the copula being thus attached to the predicate. In abolishing the 'not' of the copula, they only multiply unnecessarily the number of negative terms, and make the meaning of propositions with such terms vague and indefinite.

Some logicians make the copula consist of the present tense only of the verb 'to be' with or without the negative particle, that is, of 'is,' 'are,' 'is not,' 'are not,' and exclude the other tenses. Hamilton, Mansel, Fowler, &c., have adopted this view, while Mill contends that the element of time, or modifications of tense are as much a part of the copula as the particle 'not' or the fact of negation. Just as we cannot, he would argue, exclude the latter, so we cannot exclude the former; its exclusion from the copula would require it to be included in the predicate as in the case of the particle 'not.' As the copula expresses only a certain relation (or its absence) between the subject and the predicate, it does not consist of any particular tense. It is free from the element of time, which should be referred to the predicate.

A judgment is the mental recognition of a certain relation between two concepts. It consists of two concepts, the Subject and the Predicate, and of the recognition of a certain relation, agreement or disagreement, congruence or incongruence, &c., between them, the Copula. When the two concepts are true, that is, correspond exactly to the attributes and things actually existing, and when the relation between them is also true, that is, corresponds exactly to the relation between the attributes and things, then the judgment is true; otherwise the judgment must be regarded as false. A true judgment is the recognition of a relation existing between attributes and things. A rela-

tion between two attributes or things may be considered (1) in itself, without any reference to our thought or any mode of our thinking of it, (2) as thought by us independently of any mode of expression in language, and (3) as thought and expressed by us in language. A judgment is the relation as thought by us. A proposition is the relation as thought and expressed by us in language. By some logicians a proposition is regarded as the objective relation itself, or expressed in language without any reference to our thought or any mode of our thinking of it.

§ 2. The Divisions of Propositions.

A proposition in Logic usually corresponds to a simple or to a complex sentence in grammar, while a compound sentence in Grammar generally corresponds to a plurality of propositions in Logic.

SYMBOLICAL EXAMPLES OF PROPOSITIONS.

I. Propositions (single).

1. A is B, a simple sentence.
2. A that is C is B, a complex sentence.
3. A that is C is B that is D, a complex sentence.
4. If A is, B is, a complex sentence.
5. A is either B or C, a compound sentence.
6. If A is, either B or C is.

II. Combinations of Propositions (also called Compound Propositions).

1. A is B and C; or A is B as well as C.
2. A and D are B; or A as well as D is B.
3. A and D are B and C.
4. A that is E, and D that is F, are B.
5. A that is E, and D that is F, are B which is G.
6. A is B, and C is D.
7. A is B, but C is D.
8. A is neither B nor C.
9. Neither A nor D is C.

The various divisions of propositions are founded upon certain aspects possessed by every proposition. A tabular view of the divisions is given below :—

Propositions.	I. Relation ...	Categorical: A is B, A is not B. Conditional: If A is, B is.
	II. Quality ...	Affirmative: A is B. Negative: A is not B.
	III. Modality...	Necessary: A must be B. Assertory: A is B. Problematic: A may be B.
	IV. Quantity ...	Universal: All A is B. Particular: Some A is B.
	V. Import ...	Verbal, Analytical: All men are animals. Real, Synthetical: All men are mortal.

We shall now proceed to explain these divisions in order.

§ 3. Division of Propositions according to Relation.

The first division of propositions is into (1) Categorical and (2) Conditional, founded on the relation between the two terms, or on the nature of affirmation or denial. A categorical proposition is one in which the relation between the subject and the predicate is a simple, unconditional one, in which the predicate is simply affirmed or denied of the subject, without any condition being laid down. For example, in "A is B," "All metals are elements," B is affirmed of 'A' unconditionally, 'elements' is affirmed of 'all metals' under all circumstances without any restriction or condition. Similarly, in the proposition "Some men are wise," 'wise' is affirmed absolutely or unconditionally of 'some men.' A conditional proposition, on the other hand, is one in which the relation of dependence is affirmed or denied between two assertions or in which an affirmation or denial is made under a certain condition. In the proposition "if A is B, C is D," for example, the assertion 'C is D' depends on the assertion 'A is B,' or D is affirmed of C, provided B is affirmed of A. The truth of the second clause depends upon that of the first. Hence the first clause is called the antecedent, condition, or reason, and the second the consequent. The dependence of the one upon the other, or the conditional nature of

the affirmation in the proposition, is expressed by the word 'if' before the antecedent, and 'then' or 'therefore' understood before the consequent. The word 'if' is sometimes replaced by such words as 'when,' 'where,' 'provided that,' 'suppose,' or their equivalents. In the proposition "A is either B or C" we have conditional affirmation: 'B' is affirmed of 'A,' if 'C' is denied of 'A'; or 'C' is affirmed of 'A,' if 'B' is denied of the latter. Thus there is really one assertion, and the proposition is, in fact, equivalent to one or other of the two propositions, (1) "if A is not C, A is B"; and (2) "if A is not B, A is C."

Conditional propositions are divided into two classes, (1) Hypothetical and (2) Disjunctive, according as the two members or clauses are conjoined by 'if . . . . then,' or disjoined by 'either . . . . or.' The propositions "If A is, B is," "If A is B, C is D," "If A is, B is not," belong to the first class, and the propositions "A is either B or C," "Either A is B or C is D," &c., belong to the second class.

Disjunctive and hypothetical propositions have been also called *Complex* and even *Compound*, because they apparently consist of more than one proposition. In reality, however, they are as simple as categorical propositions, and express each but one affirmation or denial—the affirmation or the denial of the dependence of one assertion upon another, or, more properly, of one many-worded term upon another. The two clauses of a hypothetical proposition are really equivalent to two many-worded terms, and not to two categorical propositions as in the case of a compound proposition. In the proposition "If A is, B is," the antecedent 'A is' and the consequent 'B is' are not two independent assertions in which the existence of A and that of B are, respectively, affirmed, but parts of a conditional affirmation, the truth of the one part depending upon that of the other. They are, in reality, two many-worded terms, like 'that men are mortal,' 'to live happily,' &c., and mean simply 'the existence of A' and 'the existence of B' respectively; and the relation expressed by the proposition is that of dependence of the latter upon the former. Similarly, in the proposition "If A is B, C is

D," the antecedent 'A is B' means 'A being B,' 'the fact or event of A being B,' and the consequent 'C is D' means 'C being D,' or 'the fact or event of C being D'; and the relation expressed by the proposition is the dependence of the latter upon the former. The disjunctive proposition may likewise be shown to be really simple, though apparently consisting of several propositions.

According to some logicians (Hamilton, Thomson, Boole, Ueberweg, Bain, and Fowler), in a disjunctive proposition, the truth of one clause or alternative member depends on the falsity of another, and *vice versâ*. Thus in the proposition "A is either B or C," the truth of 'A is B' depends on the falsity of 'A is C,' and the falsity of 'A is B' on the truth of 'A is C'; the truth of 'A is C,' on the falsity of 'A is B,' and the falsity of 'A is C,' on the truth of 'A is B.' The disjunctive proposition "A is either B or C" is thus equivalent to one or other of the four hypothetical propositions:—

(1) If A is not C, A is B,
(2) If A is C, A is not B,
(3) If A is not B, A is C,
(4) If A is B, A is not C.

According to other logicians (Whately, Mansel, Mill, and Jevons), in a disjunctive proposition, the falsity of one alternative member implies the truth of the other, and not *vice versâ*. Thus, of the four hypotheticals above they would recognize only the first and the third, and reject the other two as not implied by the disjunctive proposition. According to them, the truth of one member does not imply the falsity of the other, and both may be true. Mill illustrates this view in the following way:— The proposition "He is either a fool or a knave" does not mean that he cannot be both a fool and a knave. Its explicit meaning is that (1) if he is not a fool, he is a knave, and that (2) if he is not a knave, he is a fool. This is, also, the view given above and seems to be the more reasonable of the two views. On the whole, however, the difference between the two views seems to be merely a verbal one. The question is, Are the two members

disjoined by 'either . . . . or' exclusive alternatives or not? If they are, then Ueberweg's view is true. If they are not, then Mill's' view is true. Which of the two is true, may be determined by usage, and it seems that usage sanctions both; sometimes the alternatives disjoined by 'either . . . or' are exclusive, and sometimes not. For example, in the propositions, "This organism is either a plant or an animal," "The soul is either mortal or immortal," the alternatives are exclusive: the same subject cannot possess the two attributes expressed by them. In the propositions, "This metal is either a conductor of electricity or a conductor of heat," "He who prefers a lower pleasure in presence of a higher, is either immoral or imprudent," "A mental phenomenon is one either of knowing, feeling, or willing," the alternatives are not exclusive: the same subject may possess the attributes expressed by them. In this book we shall recognize both the views, though preference is given to the view we have connected with Mill's name.

§ 4. Division according to Quality.

The second division of propositions is into (1) Affirmative and (2) Negative, founded on their *quality*, that is, according as the predicate is affirmed or denied of the subject. An affirmative proposition is one in which the predicate is affirmed of the subject, that is, in which the attribute signified by the predicate belongs to the subject; or in which the individual or the class denoted by the subject is included in the class denoted by the predicate; or in which there is an agreement between the ideas or notions of the subject and the predicate; or in which the attribute connoted by the predicate accompanies the attribute connoted by the subject; or lastly in which, as in the case of the hypothetical proposition, the consequent depends on the antecedent. A negative proposition, on the other hand, is one in which the attribute signified by the predicate does not belong to the subject; or in which the subject as a class is excluded from the predicate as a class; or in which there is a disagreement between the ideas of the subject and the predicate; or in which the attribute connoted by the predicate does not accompany the attribute connoted by the subject; or lastly in which, as in the

case of the hypothetical proposition, the consequent does not depend on, or is independent of, the antecedent. The quality of a categorical proposition is determined by its copula. The quality of an hypothetical proposition is determined by the quality of its consequent and is not affected in any way by the quality of its antecedent. Thus if the consequent clause of an hypothetical proposition is affirmative, the proposition is affirmative; and if it is negative, the proposition is negative.

Symbolical Examples:

A is B. If A is, B is. If A is B, C is D. If A is not B, C is D.	Affirmative.
A is not B. If A is, B is not. If A is B, C is not D. If A is not B, C is not D.	Negative.

Concrete Examples.

All metals are elements; All men are mortal. If it rains, the ground will be wet. If Hydrogen is not a non-metal, it is a metal.	Affirmative.
No men are perfect. If the wind blows from the north, it will not be hot. If a triangle is not equilateral, it is not equiangular.	Negative.

§ 5. Division according to Modality.

The third division of propositions is founded on their *modality*, and is into (1) Necessary, (2) Assertory, and (3) Problematic. The modality of a proposition is a special development of its quality. According to the latter, the predicate is affirmed or denied of the subject; on the former depends the special character of the affirmation or denial, whether the relation affirmed or denied between the subject and the predicate is a necessary, assertory, or problematic one. If the relation or connection between A and B, the subject and predicate of a proposition, be one founded on their very nature and constitution, that is, one universally and necessarily true, the modality of the proposition

is necessary: "A must be B." "The two sides of a triangle must be together greater than the third." If the connection be one established by experience, and true as far as experience extends, that is, one not implying necessity, the modality of the proposition is assertory: "A is B"; "All men are mortal"; "All material bodies gravitate." If the connection be uncertain, true under certain circumstances, and not under others, if A may or may not be B, then the modality of the proposition is said to be problematic; as in the propositions "It may rain to-morrow," "He may be wise," "He is probably a good man." The modality of a proposition thus consists in the degree of necessity, certainty, or probability of the connection or relation between the subject and the predicate, and is expressed by such words as must be, necessarily, certainly, most probably, probably, may be, &c.

Dr Venn, in his work on the *Logic of Chance*, argues that modal propositions cannot be satisfactorily treated of in Pure Logic, or the Logic of Certainty, but only in the Logic of Probability[1]. Hamilton, Mansel, and others exclude modality from Logic. Hamilton excludes it altogether from logical propositions. Fowler confines it to the predicate and keeps the copula free from all adverbs of time, place, &c., as well as from all words and phrases expressive of the degrees of conviction or certainty. Ueberweg, following Aristotle, gives three kinds or varieties of modality:—(1) Necessary or Universal: A must be B. (2) Assertory: A is B. (3) Contingent or Problematic: A may be B. Dr Venn maintains that assertory and necessary propositions express the same full belief or conviction, while problematic propositions express all the degrees of conviction, so that the division is really into two and not three distinct classes. This subject need not be discussed here: but the question is, Is the certainty or the mental conviction of propositions such as "all the three angles of a triangle are together equal to two right angles," of the same kind and degree as that of propositions like "all men are mortal," "all material bodies gravitate"?

[1] See below, the Chapter on "Probable Reasoning and Probability."

§ 6. Division according to Quantity.

The fourth division of propositions is into (1) Universal, and (2) Particular, founded on their *quantity*. A categorical proposition is universal or particular according as its subject is taken in its entire or in its partial extent. Its quantity is determined by the quantity of its subject. "All A is B" and "No A is B" are both universal, because, in the former, 'B' is affirmed, and, in the latter, B is denied, of the whole of 'A,' that is, of every individual thing denoted by 'A.' "Some A is B," "some A is not B" are both particular, because, in the former, B is affirmed, and, in the latter, B is denied, of a part of the subject 'A.' The logical meaning of the proposition "Some A is B" is that 'at least one A is B,' that 'B' is affirmed of at least one individual, if not of more, belonging to the class 'A.'

A proposition of the form "A is B" or "A is not B" is said to be an *indesignate* or *indefinite* proposition, because its quantity, or rather the quantity of its subject, is not stated explicitly; the propositions "Metals conduct electricity," "animals have a nervous system," "plants have flowers," "material bodies have weight," &c., belong to this class. The quantity of these propositions cannot be determined without a reference to the sciences to which they respectively belong; or, in other words, without a knowledge of their *matter* as distinguished from their *form;* but it is, in reality, either universal or particular, that is, the predicate in each of them is affirmed either of the whole of the subject, or of at least a part of it—of every individual thing denoted by it, or of at least one individual.

When the subject of a proposition consists of a general term such as 'a German,' 'a man,' or of a singular term such as 'Plato,' 'this man,' the proposition is called by some Logicians *Singular*. Such a proposition should be referred to the class of universal propositions, when the subject denotes definitely an individual or a collection of individuals, as the predicate is, in that case, affirmed of the whole of the subject; and to the class of particular propositions, when the subject does not definitely refer to an individual or a collection of individuals. For example,

the proposition "A man was there" is a singular proposition belonging to the class of particular propositions, while the proposition "This man was there" is a singular proposition, belonging to the class of universal propositions. "One metal is liquid" is a singular proposition belonging to the former class, while "Mercury is a liquid metal" is a singular proposition belonging to the latter class. In like manner, when by any descriptive words, or demonstrative pronouns, any individuals of a class forming the subject of a proposition are definitely pointed out, the proposition is universal and not particular: "These three men were there," "These metals belong to the Copper Group,' "All metals except mercury are solid substances," "Those metals that do not rust are noble metals," are all universal propositions.

We have explained above the quantity of categorical propositions, when the subject is taken in its denotation or extent. We get the same two-fold division, when the subject is taken in its connotation or intension, for the attribute signified by the predicate B may accompany the attribute connoted by the subject A in every case, or in some cases,—under all circumstances universally, or under particular circumstances contingently. In the former case, the proposition "A is B" is universal, and in the latter case, it is particular. For example, the proposition "All men are mortal" is universal, and means, when the subject is taken in its connotation, that mortality accompanies humanity under all circumstances, that wherever humanity is, mortality is. The proposition "Some men are wise" is particular, and means, when the subject is taken in its connotation, that in some cases, or under certain circumstances, wisdom accompanies humanity, that in at least one case, where humanity is, wisdom is.

The hypothetical proposition is universal, when, in every case, the antecedent is followed by the consequent; and it is particular, when the consequent follows the antecedent in some cases, or in at least one case. The universal proposition "If A is, B is," or, more explicitly, "In all cases, if A is, B is," means that wherever 'A' exists 'B' exists, that under whatever circum-

stances 'A' happens, it is followed by the happening of 'B'; and the particular proposition "In some cases, if A is, B is,' means that, in at least one case, the existence of 'A' is followed by the existence of 'B.'

EXAMPLES.

I. Universal.

1. All men are mortal.
2. No man is perfect.
3. If mercury is heated, it rises in temperature.
4. If water is heated to 100° C. under a pressure of 760 mm., it boils.
5. This animal is either a vertebrate, or an invertebrate.
6. The soul is either mortal or immortal.
7. Space is either finite or infinite.

II. Particular.

1. Some men are wise.
2. Some elements are not metals.
3. In some cases, if water is heated, it contracts.
4. In many cases, if there is a sensation, there is a perception.
5. In some cases, if there is a sensation, there is no perception.
6. Some men are either philosophers or prophets.

§ 7. The Propositional Forms according to Quality and Quantity.

Propositions are divided into affirmative and negative according to their quality. The affirmative propositions, as well as the negative, may again be divided into universal and particular according to their quantity. Thus we get the following classes or forms of propositions :—

PROPOSITIONS

Affirmative		Negative	
Universal	**Particular**	**Universal**	**Particular**
All A is B; In all cases, if A is, B is.	**Some A is B; In some cases, if A is, B is.**	**No A is B; In all cases, if A is, B is not.**	**Some A is not B; In some cases, if A is, B is not.**
A	**I**	**E**	**O**

Every universal affirmative proposition is called A, every universal negative proposition E, every particular affirmative proposition I, and every particular negative O, that is, A, E, I, and O are the symbols for the propositions of those classes respectively. The words 'all,' 'the whole,' 'any,' 'each,' 'every,' 'a few' and 'certain' used definitely, 'no,' 'none,' &c., are signs of A or E. The words 'some,' 'not all,' 'at least one,' 'not none,' 'a few' and 'certain' used indefinitely, 'many,' 'most,' &c., are signs of I or O.

The quality and quantity of a proposition cannot always be determined from its form. Without a knowledge of the subject-matter, we cannot, in many cases, say whether it is universal or particular, affirmative or negative. For example, the proposition "Every man is not learned" would seem to be E from its *form*, but from its *meaning* it is really O or I, that is, it means that some men are not learned, and implies that some men are. Thus it may be taken, from its meaning, to be indifferently O or I; but in Logic, it is usually regarded as a mere negation of the proposition "All men are learned," and treated as O rather than as I. Similarly, the propositions "Every mistake is not a proof of ignorance," "Some of the most valuable books are seldom read," "Few know both physics and metaphysics," "All that glitters is not gold," "All elements are not metals," "All scientific books are not difficult," are to be regarded as O, rather than as I. The proposition "Some acids have no oxygen" would seem from its form to be affirmative, "having no oxygen" being affirmed of some acids, but, in reality, it is negative, and means that 'having oxygen' is denied of some acids. Similarly, "None were there," "Nothing is annihilated," "Many objects of imagination have no objective existence," should be regarded as negative rather than as affirmative.

Similarly, the modality of a proposition cannot, in every case, be determined from its form only. For example, the proposition "All triangles have three angles together equal to two right angles" would appear from its form to be assertory, but, in reality, it is necessary.

Exercise.

Reduce each of the following propositions to the logical form, and give its quantity and quality, that is, state in respect to each whether it is A, E, I, or O :—

(1) Two straight lines cannot inclose a space.
(2) Matter is anything whose existence can be determined by one or more of our senses.
(3) A nail driven into wood is not a true case of penetration.
(4) Liquids have no shape of their own.
(5) Gases are eminently compressible and expansive.
(6) Strictly speaking, impenetrability only applies to the atoms of bodies.
(7) Two portions of matter cannot simultaneously occupy the same portion of space.
(8) If a pint of water and a pint of alcohol be mixed together, the volume of the mixture is less than two parts.
(9) Very few of these elements occur in nature in the free state.
(10) No absolute rest is known in the universe.
(11) Inertia is a purely negative property of matter.
(12) Consciousness involves judgment.
(13) The province of physics is at present much more restricted.
(14) To have the objective essence of a thing is to think clearly what is in it and omit what is not.
(15) Not all our ideas consist of the objective essences of things.
(16) Some of our ideas represent only the partial or accidental affections of things.
(17) If you know what a circle is, and what a square, you cannot make a compound out of them.

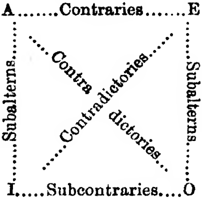
§ 8. The mutual relations of A, E, I, and O, or, Opposition of Propositions.

Two propositions having the same subject and predicate, but differing in quality, are said to be opposed to each other, and their mutual relation is called opposition.

The relation of *A* and *E* to each other is called Contrary Opposition. That is, two universal propositions having the same subject and predicate, but differing in quality, are said to be

contrarily opposed to each other, and their mutual relation is called *Contrary Opposition.*

The relation of A and O to each other, as well as that of E and I to each other, is called Contradictory Opposition. That is, two propositions having the same subject and predicate, but differing both in quality and quantity, are said to be *contradictorily opposed* to each other, and their mutual relation is called *Contradictory Opposition*.



The relation of I and O to each other is called Subcontrary Opposition. That is, two particular propositions having the same subject and predicate, but differing in quality, are said to be *subcontrarily opposed* to each other, and their mutual relation is called *Subcontrary Opposition.*

The relation of A and I to each other, as well as that of E and O to each other, is called Subalternation. That is, two propositions having the same subject and predicate, and the same quality, but differing in quantity, are said to bear to each other the relation of *subalternation;* the one of universal quantity is called the *subalternant*, and the other of particular quantity the *Subalternate;* and both are called *Subalterns.*

The Opposition of Propositions is, therefore, of three kinds; (1) Contrary; (2) Contradictory, and (3) Subcontrary. Subalternation is, also, sometimes called a kind of opposition; but there is no opposition between the subalternant and the subalternate, both of which have the same quality and differ in quantity only.

Exercise.

Give the contradictory, the contrary or subcontrary, and the subalternant or subalternate, of the following propositions :—

(1) Every metal conducts heat.

(2) Every planet moves round the sun.

(3) Matter cannot change its own state of motion or of rest.

(4) All plants have not flowers.
(5) Some elements are not metals.
(6) All material bodies are extended.
(7) Heat expands bodies.
(8) Gold is a metal.
(9) A sensation can only be in a sentient being.
(10) Gases and liquids are perfectly elastic.
(11) Liquids have no shape of their own.
(12) Consciousness is an immediate knowledge.
(13) In nature, relative motion and rest are alone presented to our observation.
(14) If all impeding causes were removed, a body once in motion would continue to move for ever.
(15) Water sometimes contracts by heat.
(16) A sensation is sometimes not accompanied by a perception.

§ 9. Division according to Import[1].

The last division of propositions, which we need notice, is founded on the relation of the connotation of the predicate to that of the subject, or, in other words, on the old distinction of Essential and Accidental Predication, and is into (1) Verbal, Analytical, Essential, or Explicative, and (2) Real, Synthetical, Accidental, or Ampliative. When the connotation of the predicate of a proposition is the same as, or a part of, the connotation of the subject, the proposition is called Verbal or Analytical. When, on the other hand, the connotation of the predicate is not a part of that of the subject, the proposition is called Real or Synthetical. In the former case, the predicate merely explains,

[1] The division of propositions into (1) Verbal, Analytical, &c., and (2) Real, Synthetical, &c., is here given as founded on their *import*, for the meaning or import of a proposition is different according as it belongs to one or the other of the two classes. It may also be regarded as founded on the *mode of their formation;* for an analytical proposition may be regarded as formed by the analysis or resolution into parts of the connotation of the subject, and a synthetical proposition by the synthesis or union of the connotations of the subject and the predicate.

or states the entire meaning, or a part of the meaning, of the subject; and the proposition imparts no new information to those who already know the meaning of the subject. In the latter case, the proposition imparts new information, and the attribute connoted by the predicate is a real addition to that connoted by the subject. Thus the proposition "All men are rational" is verbal, because the attribute 'rationality' is a part of the larger attribute or group of attributes 'humanity,' while the proposition "All men are mortal" is real, because the attribute 'mortality' is not contained in the connotation of the subject 'man'; it is something different from, and new to, humanity; and the proposition expresses the conjunction of these two attributes.

§ 10. The Five Predicables: Genus, Species, Differentia, Proprium, and Accidens:—In a verbal proposition, the predicate, in relation to the subject, is either a genus, a species, or a differentia. In a real proposition, the predicate, in relation to the subject, is either a proprium, or an accidens. In other words, if the predicate of a proposition, in relation to the subject, be a genus, species, or differentia, the proposition is verbal, that is, the connotation of the predicate must be a part of that of the subject. If, on the other hand, the predicate be a proprium, or an accidens, the proposition is real, that is, the connotation of the predicate is not contained in that of the subject.

If the subject of a verbal proposition be an individual, the predicate, in relation to the subject, is called a species. If the subject be a class, the predicate, in relation to it, is called a genus, and the subject, in relation to the predicate, a species. The two terms, genus and species, are thus entirely relative to each other, and one has a meaning only in relation to the other. Given two terms related to each other as genus and species, the connotation of the latter minus the connotation of the former is equivalent to the differentia of the species, that is, to the attribute or group of attributes, which distinguishes that species from others belonging to the same genus. Thus the three terms genus, species, and differentia, implying each the other two, are correlatives. Further, just as a genus implies that there are

species under it, so a species implies that there are individuals under it. And so to the three correlatives mentioned above may be added another, namely, Individual; and the four terms may be thus defined :—

A Genus in extension is a class including smaller classes or species. In comprehension, it is included in the species.

A Species in extension is a class which is included in a larger class or genus, and which includes individuals. In comprehension, it includes the genus and the differentia, and is included in the individual.

A Differentia is an attribute which distinguishes one species from others belonging to the same genus, and which is included in the comprehension of the species. In extension, that is, when taken as a general term, it includes the species and the individual.

An Individual is a particular thing or substance having a unique group of attributes, partly known and partly unknown. In extension, it is included in the species. In comprehension, it includes the species, and consequently also the differentia and genus.

The connotation of a species = the connotation of the genus + the differentia.

∴ The differentia = the connotation of the species − the connotation of the genus; and the connotation of the genus = the connotation of the species − the differentia. Example :—Taking 'animal' and 'man' as genus and species we have :—

Humanity = Animality + the Differentia; ∴ the Differentia = Humanity − Animality.

But, Humanity = Animality + Rationality, that is, the connotation of man consists of those two attributes.

∴ The Differentia = Animality + Rationality − Animality = Rationality. That is, 'rationality' is the differentia of the species 'man' in the relation to the genus 'animal,' and by the attribute 'rationality' the species 'man' is distinguished from others belonging to the same genus 'animal.' Similarly, an individual may have its differentia which is equivalent to the connotation of the individual minus that of the species :—

Socraticity = Humanity + the Differentia ; ∴ the Differentia of Socrates = Socraticity − Humanity.

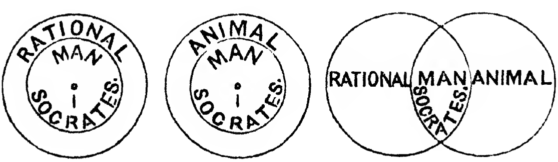
By the 'Differentia of Socrates' is meant the group of attributes by which he is distinguished from other individuals belonging to the same species 'man.'

The differentia of a genus, like that of a species, in reference to a higher class, is the connotation of the genus minus the connotation of the higher class. Thus the differentia of 'animal' in relation to the higher class 'organic being' = animality − the attribute of being organised; or sentiency; animal being defined as a sentient organized being.

In extension, a species is included in the genus, and an individual in the species. Thus 'animal' contains 'man'; and 'man' contains 'Socrates'; 'metal' contains 'gold'; 'organism' contains 'animal.' A differentia, when taken in extension, is a larger whole than the species. Sometimes, however, it coincides with the extension of the species; but the comprehension of the differentia being smaller than that of the species, its extent is theoretically greater than that of the latter.

The relation of individual, differentia, species, and genus may thus be represented by diagrams:

1 2 3



The dot in the centre stands for Socrates. The inner circle for man. The outer circle for rational in the first diagram, and for animal in the second, the relation of animal and rational is shown in the third.

A *Proprium* (or property) of a genus, species, or individual is any attribute which follows from its comprehension either deductively or causally. If it follows from the comprehension of the genus, the property is called *generic;* if from that of the species, *specific;* and if from that of the individual object, *individual.* Thus, an individual thing may have its individual property, its specific property, or a property following from the species to which the individual belongs, and even a generic property following from the genus to which its species belongs. This last may be included in the specific property. A species may have two properties, one following from its differentia, and the other from its genus. The former is called the specific, and the latter the generic property, of the species; or both together are simply called its *property.* 'Memory,' for example, may be regarded as a property of man, following either from the genus animal, or from the differentia rational; 'power of judging' is likewise a property of man following from the differentia. The properties of the triangle, as proved in the *Elements* of Euclid, follow partly from the comprehension of its genus figure, partly from that of triangle, and partly from those of special kinds of triangles.

An *Accidens* (or accident) of an individual, genus, or species is any attribute which is possessed by it, and which does not follow from, or form a part of, its comprehension. If an accidens always belongs to an individual, or if it belongs to all the members of a genus, or species, it is called an *inseparable accidens* of that individual, genus, or species; as the place or date of birth of a particular person, the hair of man, the blackness of the crow, the whiteness of snow, &c. If, on the other hand, an accidens is sometimes present and sometimes absent in an individual, or if it belongs to a part only of a species or genus, then it is called a *separable accidens* of that individual, species, or genus; as the walking or sitting of a particular person, the wisdom of man, the solubility in water of salts, the opacity of gases, the learning of man, &c.

When the predicate of a proposition is a proprium, or an

accidens, of the subject, the latter in extension is included in the former, that is, the extension of the accidens or proprium, when taken as a general term, is a greater whole than that of the subject; while, in comprehension, the predicate expresses an attribute not contained in the connotation of the subject, that is, it imparts some new information about it; and the proposition, therefore, belongs to the class of real. In the proposition "Water boils at 100° C., under a pressure of 760 mm.," the attribute expressed by the predicate is not a part of the connotation of the term water.

The five terms—genus, species, differentia, proprium, and accidens—are called predicables, because whatever may be predicated (affirmed) of a subject in a proposition is, in relation to the subject, one or other of the five. A predicable is thus a name of a class of predicates in relation to the subjects. It should be distinguished, on the one hand, from the word 'predicament,' or 'category,' which means a most general class of both subjects and predicates, and, on the other, from the word 'predicate,' which means what is affirmed or denied of a subject. Given a term: whatever be affirmed of it, the predicate, *in relation* to the subject, is a predicable, that is, it is either a genus, species, differentia, proprium, or accidens; and the subject as well as the predicate must belong to some category or other. Aristotle gave four predicables, *viz.*, genus, definition, proprium, and accidens. Later logicians added 'species' and 'differentia' to Aristotle's list, and removed 'definition' from it. Thus there came to be the five predicables we have explained above. Some logicians have made further additions to the list. Professor Fowler, for example, gives 'synonym,' 'definition,' 'designation,' *idion* (a Greek word signifying a peculiar property), in addition to the five, while others regard them as falling under one or other of the five predicables adopted by them: 'synonym' and 'designation,' for example, would be regarded by some of them as included in accidens, 'definition' as a compound of genus and differentia, and 'idion' as coming under either differentia or property.

Besides the terms explained above, the older logicians use the term *summum genus* to mean a highest genus or a genus which cannot be a species, being the highest and most general of its kind, and the term *infima species* to mean a lowest species or a class which cannot be a genus to another, being the lowest of its kind, while the intermediate genera and species are called by them *subaltern genera* and *species*. 'Substance,' for example, is regarded by them as a summum genus, 'man' as an infima species, incapable of further subdivision into species, and 'body,' 'living being,' and 'animal' as subaltern genera and species.

The two terms 'genus' and 'species' express the relation of containing and contained. Any class containing another is popularly called a genus in relation to the latter, which is called a species. In the Sciences of Classification, in Botany and Zoology, for example, groups of a particular description are called genera in relation to others of an equally definite nature, which are called species. In order to express the relation of containing and contained, we not only use the two old terms, genus and species, but also many others according to the position of the groups in a system of division or classification. For example, the terms kingdom and sub-kingdom, class and sub-class, order and sub-order, genus and sub-genus, species and sub-species, variety and sub-variety, used in Zoology and Botany, mark as clearly the relation of containing and contained as the two words, genus and species.

Exercises.

I. State whether the following propositions are verbal or real, analytical or synthetical, and whether the predicate in relation to the subject is a genus, species, differentia, proprium, or accidens:—

1. Oxygen is an elementary gas.
2. Water boils at 100° C., under a pressure of 760 mm.
3. Platinum is a rare metal.
4. Sugar is sweet.
5. The atmospheric air is a mixture of nitrogen and oxygen.

6. Copper conducts heat as well as electricity.
7. All men have the power of thinking.
8. All animals are sentient beings.
9. All the flowering plants have fruits.
10. Heat expands bodies.
11. The leaves of plants are green.
12. Spring-water contains many salts in solution.
13. Hydrogen is the lightest substance known.
14. London is the largest city in England.
15. Milton was blind when he composed the "Paradise Lost."

II. Give the genus, species, differentia, proprium, and accidens of each of the following terms:—

(1) Triangle, (2) Circle, (3) Straight line, (4) Square, (5) Right angle, (6) Element, (7) Force, (8) Material Body, (9) Animal, (10) Chalk, (11) Rock, (12) Virtue, (13) Volition, (14) Knowledge, (15) Pleasure.

§ 11. Miscellaneous Exercises on Propositions.

In describing the logical characters of a proposition, the following method should be followed:—

I. What is given is a sentence. Ascertain whether the sentence consists of a single proposition or of a plurality of propositions.

II. In the former case, state whether it is—

- i. Categorical, Hypothetical, or Disjunctive.
- ii. Affirmative or Negative.
- iii. Necessary, Assertory, or Problematic.
- iv. Universal, Particular, or Indesignate; Singular and Universal, or Singular and Particular.
- v. Verbal (or Analytical) or Real (or Synthetical).

Both the quality and quantity of a proposition may also be stated at once by saying whether it is A, E, I, or O.

III. In the latter case, state the propositions of which it consists, and treat each of them as detailed above.

IV. Sometimes the quality, quantity, and other characters of a proposition are not quite evident from its form or the manner of its statement. In such cases, verbal changes should be made in order to state it in the logical form, keeping the meaning the same. It is always safe first to ascertain, as in the case of the term, the meaning

of the proposition, or, where this is not practicable, to see, before attempting to describe the logical characters of the proposition, whether the subject be a general term taken distributively or not, whether there be any negative particle attached to the copula or to the predicate, whether there are any signs of universality or negation before the subject, &c.

Examples.

1. "No man is perfect": categorical, negative, assertory, universal, and real.

2. "The three angles of a triangle are together equal to two right angles": categorical, affirmative, assertory in form, but really necessary, universal, and real.

3. "Some elements are not metals": categorical, negative, assertory, particular, and real.

4. "None but material bodies have weight": this proposition really means that "all things having weight are material bodies." In this form it is an A proposition. In the original form, it may be regarded as an E proposition, "no not-material bodies have weight," signifying that having weight is denied of all things other than, or except, material bodies, that none that have weight are other than material bodies, and this last is the same as "all things having weight are material bodies," the proposition we have substituted above for the original one. It should be noted that the proposition does not mean that every material body has weight.

5. "All metals except mercury are solids."—In this proposition 'solids' is affirmed of all metals except mercury, and the proposition may, therefore, be regarded as an A proposition and described as categorical, affirmative, assertory, universal, and real. Or it may be taken as an I proposition, 'some metals are solids,' but in this degraded form, the full meaning of the original proposition is not expressed. Or we might state the names of all the metals except mercury, and form a proposition with them all as the subject and 'solids' as the predicate as before. For example, 'gold, copper, iron, silver, &c., are solids.' Such a proposition would be a combination of the several propositions, having each a certain metal for its subject, and 'is a solid' for its copula and predicate. Thus, 'gold is a solid,' 'copper is a solid,' 'iron is a solid,' and so forth.

6. "All is not gold that glitters," = "All that glitters is not gold." This proposition is really O, though it has the form of E. It really means that at least some thing that glitters is not gold.

7. "If mercury be heated, it will expand": conditional, affirmative, assertory, universal, real.

8. "All men are rational, but all are not wise": this sentence is a combination of the two propositions—(1) 'All men are rational' (A), and (2) 'All men are not wise' (O).

9. "Gravity as well as heat can produce motion": a combination of the two propositions, (1) 'Gravity can produce motion' (A), and (2) 'Heat can produce motion' (A).

Examples for Solution.

Treat the propositions[1] given below as follows:—

I.—Describe the logical characters of each of them.

II.—Give the contradictory, the contrary or subcontrary, and the subalternant or subalternate of each of them.

III.—State the relation of the predicate to the subject in each of the affirmative propositions.

IV.—In the case of a disjunctive proposition, state the hypothetical propositions, one or other of which is equivalent to it.

1. Every pure substance consists of similar molecules.
2. Some animals have no power of locomotion.
3. Sensations are passive states of the mind.
4. Nothing is annihilated.
5. All metals except one are solid.
6. Benevolence is a virtue.
7. Only the virtuous are happy.
8. Certain metals are ductile.
9. Some substances have no cause.
10. Uneasy rests the head that wears a crown.

[1] Most of the propositions given here are taken from Ganot's *Popular Natural Philosophy*, Roscoe's *Chemistry*, and Reid's *Inquiry*, exactly in the form in which they are expressed by the authors. They are kept in that form in order that students may acquire the habit of describing the characters of propositions as they actually occur in the works of authors, instead of the contracted and artificial propositions of the Logician.

11. None were there.
12. None but sensations can resemble sensations.
13. Metals conduct heat and electricity.
14. Oxygen is a colourless, invisible gas, possessing neither taste nor smell.
15. Hydrogen is the lightest body known.
16. Matter is indestructible.
17. Most of the acids are soluble in water.
18. All acids contain hydrogen and always contain also oxygen.
19. The passage of water to the state of ice, and the return of the latter to the liquid state, are physical phenomena.
20. The mass of a body is the quantity of matter contained in the body.
21. The elementary atoms can unite with each other to form compounds, but cannot be destroyed by any known process.
22. If molecular attraction were the only force acting upon the small particles of which bodies are composed, they would come into complete contact.
23. All bodies are extended, impenetrable, divisible, porous, compressible, and elastic.
24. Strictly speaking, impenetrability only applies to the atoms of bodies.
25. Divisibility, porosity, compressibility, and elasticity do not apply to atoms, but only to bodies or aggregates of atoms.
26. Two portions of matter cannot simultaneously occupy the same portion of space.
27. Compressibility is both a consequence and a proof of porosity.
28. Both rest and motion are either absolute or relative.
29. Bodies are either opaque or transparent.
30. If a small quantity of manganese di-oxide be mixed with the potassium chlorate, the oxygen is given off from the chlorate at a much lower temperature.
31. Oxygen can be prepared by heating powdered potassium chlorate in a small thin glass flask.
32. All the elements with the single exception of fluorine combine with oxygen to form oxides.
33. Sulphur exists in three modifications.
34. Many organic bodies are completely decomposed and charred by strong sulphuric acid.

35. Phosphorus does not dissolve in water, alcohol, or ether.
36. Arsenic is sometimes found in the free state, but more frequently combined chiefly with iron, nickel, cobalt, and sulphur.
37. Truly these ideas seem to be very capricious in their agreements and disagreements.
38. Motion is either rectilinear or curvilinear.
39. Each kind of motion is either uniform or varied.
40. Matter cannot change its own state of motion or of rest.
41. A power is a force which tends to produce motion.
42. The surfaces of bodies are never perfectly smooth.
43. Without friction on the ground neither man nor animals, neither ordinary carriages nor railway ones, could move.
44. If all impeding causes were removed, a body once in motion would continue to move for ever.
45. Some brutes are sensible of honor and disgrace.
46. Hardness and softness are neither sensations, nor like any sensations.
47. A sensation can only be in a sentient being.
48. No man can conceive any sensation to resemble any known qualities of bodies.
49. If we trust to the conjectures of men of great genius in the operation of nature, we have only the chance of going wrong in an ingenuous manner.
50. If dry chlorine gas be passed over silver nitrate, silver chloride is formed, oxygen is given off, and a white crystalline substance produced, which, on analysis, is found to be nitrogen peroxide.
51. If nitrogen monoxide gas (or laughing gas) be brought under a pressure of about 30 atmospheres at 0° C. or if it be cooled down to -86° C. under the ordinary pressure, it forms a colourless liquid.
52. If this liquid be cooled below -115° C., it solidifies to a transparent mass.
53. If carbon were not present in the earth, no single vegetable or animal body such as we know could exist.
54. If a piece of lime be held in the oxyhydrogen flame, it becomes strongly heated and gives off intense light.

55. The ignition of phosphorus takes place by slight friction, or by a blow, and even the heat of the hand may cause this substance to ignite.
56. The number of the metals is much larger than that of the non-metals.
57. The atmosphere is the gaseous envelope encircling the earth.
58. If a series of electric discharges be passed through pure oxygen, the gas becomes diminished in volume by about one-twelfth, and is partly transformed into ozone.
59. If we would know the works of God, we must consult themselves with attention and humility.
60. I know that I know.
61. Consciousness is an actual and not a potential knowledge.
62. If mediate knowledge be in propriety a knowledge, consciousness is not co-extensive with knowledge.
63. Where two, three, or more mental states are confounded, we are conscious of them as one.
64. Without memory our mental states could not be held fast, compared, distinguished from each other, and referred to self.
65. The theory of ideas is, indeed, very ancient, and hath been very universally received.
66. Common sense holds nothing of philosophy, nor needs her aid.
67. To attend accurately to the operations of our mind, and make them an object of thought, is no easy matter, even to the contemplative, and to the bulk of mankind is next to impossible.
68. He must either be a fool, or want to make a fool of me, that would reason me out of my reason and senses.
69. If philosophy contradicts herself, befools her votaries, and deprives them of every object worthy to be pursued or enjoyed, let her be sent to the infernal regions from which she must have had her origin.
70. To reason against any of these kinds of evidence is absurd, nay to reason for them is absurd.
71. We must either admit the conclusion or call in question the premises.
72. Ideas seem to have something in their nature unfriendly to other existences.

73. If one set of ideas makes a covenant, another breaks it, and a third is punished for it, there is reason to think that justice is no natural virtue in the ideal system.
74. The smell of a rose is a certain affection or feeling of the mind.
75. Some tastes and smells stimulate the nerves and raise the spirit.
76. That such a noise is in the street, such another in the room about me; that this is a knock at my door, that a person walking upstairs,—is probably learned by experience.
77. The parallelism of the eyes in general is the work of nature.
78. If a man hath lost the sight of one eye, he very often loses the habit of directing it exactly to the object he looks at.
79. A miniature painter or an engraver sees very near objects better than a sailor.
80. That we see objects single with two eyes, as well as that we see objects erect by inverted images, is attributed by Bishop Berkeley and Dr Smith entirely to custom.
81. If two visible appearances have the same visible place, they are incapable of distinction, and we see the objects single or one object only.
82. A just interpretation of nature is the only sound and orthodox philosophy.

CHAPTER II.

THE THEORY OF PREDICATION AND THE IMPORT OF PROPOSITIONS.

§ 1. WHAT is the import or meaning of a proposition or predication? What is the thought or fact expressed by it? What is the signification of its subject, of its predicate, and of its copula? In other words, in all propositions or predications of the type "A is B" (or "A is not B"), what is A, what is B, and what is the relation between them? A consistent answer to this question is a theory of Predication and of the import of Propositions. On this most important subject, there is great difference of opinion among logicians. It is proposed to give here an account of their views, as far as possible, in their own language and from their own point of view.

§ 2. I. The natural view seems to be that 'B' is an attribute, and that this attribute is referred or said to belong to the objects denoted by 'A,' as in the proposition 'Snow is white,' 'whiteness' is said to belong to the thing called 'snow.' This view is thus explained and defended by Dr James Martineau: "In saying 'Birds are warm-blooded,' we neither think of class within class, nor of attribute within attribute: the word 'warm-blooded' represents to us no conception of a *genus;* it is not a name, but a mere attributive. The word 'birds' expresses to us *no attribute,* as such; it is not a mere attributive, but a name. The term in the predicate acts upon the mind by its connotation, or in its comprehension; the term in the subject, by its denotation or in its extension; and the foregoing sentence has its

import in this,—that we refer the attribute 'warm-blood' to the class of objects 'birds.' Hence it is that, while a purely connotative word (an adjective) is all that is required in the predicate, a denotative term is indispensable in the subject.......The mind predicates nothing except about substantive objects of thought; and of them (in the class of propositions now under consideration) it predicates nothing but attributes[1]." According to Dr Martineau, the Denotative or Class Theory of Predication and Mill's Connotative Theory are both psychologically false.

All propositions do not, according to Dr Martineau, express the relation of substance and attribute. There are classes of propositions which express other relations. "The notion of substance and attribute, with the relations of genera and species to which it introduces us, is but one......of several categories of thought." "It is the basis of all class-reasoning, and supplies the common logical canon of necessity, that 'what is true of the containing is true of the contained.'" But all Demonstrative Reasoning should not be forced into this single type. There are other types of Demonstrative Reasoning founded upon other relations expressed by propositions. Propositions may, for example, express the relations of time and space, of cause and effect, of resemblance and difference, and give rise to types of Demonstrative Reasoning quite distinct from that of class-reasoning. "The attempt," says Martineau, "to coerce all reasoning into this single type—comprehensive as it is—appears to us arbitrary in itself,—and precluded from success except on condition of much violent psychology. The ideas of space and time, of cause and effect, of resemblance and difference, seem to involve distinct laws of thought, to create for themselves special elements and functions of language, and to require separate canons of Logic."

According to Martineau, therefore, there are different classes of propositions expressing different categories of thought, and there are as many distinct types of Demonstrative Reasoning as

[1] *Essays*, Vol. II. p. 351.

there are fundamental laws of thought arising from these categories.

§ 3. II. Hamilton's view :—

"To judge is to recognize the relation of congruence or of confliction, in which two concepts, two individual things, or a concept and an individual, compared together, stand to each other. This recognition considered as an internal consciousness, is called a Judgment, considered as expressed in language, it is called a Proposition or Predication." This definition is then explained. "When two or more thoughts are given in consciousness, there is in general an endeavour on our part to discover in them and to develop a relation of congruence or of confliction, that is, we endeavour to find out whether these thoughts will or will not coincide,—may or may not be blended into one; if they coincide, we judge, we enounce their congruence or compatibility: if they do not coincide, we judge, we enounce their confliction or incompatibility. Thus, if we compare the thoughts, water, iron, and rusting, we find them congruent, and connect them into a single thought, thus, water rusts iron; in that case we form a judgment[1]." Hamilton finally defines a judgment as follows: "We may, therefore, articulately define a judgment or proposition to be the product of that act in which we pronounce that of two notions thought as subject and as predicate, *the one does or does not constitute a part of the other*, either in the quantity of extension, or in the quantity of comprehension[2]."

According to Hamilton, therefore, 'A' and 'B' in the typical judgment 'A is B' are two concepts, the one forming a part of the other. From what he says elsewhere, we know he maintains that in the quantity of comprehension, 'B' is a part of 'A,' and that in the quantity of extension, 'A' is a part of 'B.' That is, the proposition has a two-fold meaning according as you take the two concepts 'A' and 'B' in their comprehension or in their extension. When 'A' and 'B' are taken in their comprehension, the meaning of the proposition is that the elementary notions constituting the concept 'B' are a part of those constituting the

[1] Hamilton's *Lectures*, Vol. III. pp. 226—7. [2] *Ibid.* p. 229.

concept 'A'; and when they are taken in extension, the meaning is that the individual things or objects included in the extension of 'A' are a part of those included in the extension of 'B.'

§ 4. III. Mansel's view:—

"When I assert that A is B, I do not mean that the attributes constituting the concept A are identical with those constituting the concept B; for this is only true in identical judgments; but that the object in which the one set of attributes is found is the same as that in which the other set is found." For example, "when I assert that the rose is fragrant, I imply that the thing which affects in a certain manner my power of sight, is in some manner identical with that which affects in a certain way my power of smell." Mansel thus defines a concept and a judgment: "A concept is a collection of attributes united by a sign, and representing a possible object of intuition." "A judgment is a combination of two concepts, related to one or more common objects of possible intuition." "The subjects of all logical judgments which are to be distinguished from the psychological, such as the spontaneous judgments of perceptive and imaginative faculties, are concepts[1]."

According to Mansel, therefore, 'A' and 'B' are both concepts, and the meaning of the proposition (when not identical) is that the attributes signified by both 'A' and 'B' exist in the same object or objects.

§ 5. IV. Ueberweg's view:—

"The judgment is the consciousness of the objective validity of a subjective union of conceptions, whose forms are different, but belong to each other. It is the consciousness, whether or not the analogous combination exists between the corresponding objective elements. As the individual conception corresponds to the individual existence, so the judgment in its various forms corresponds to, and is the subjective copy of, the various objective relations. A judgment expressed in words is an assertion or proposition[2]."

[1] *Prolegomena Logica*, 2nd edition, 1860, pp. 67—69.

[2] Ueberweg's *Logic*, p. 187.

According to Ueberweg, therefore, 'A' and 'B' are two conceptions or concepts, and the meaning of the judgment 'A is B' is that, corresponding to the union of the two concepts, there is an objective union. In other words, a mere combination of conceptions is not a judgment; but there must be the conviction that the combination has objective validity.

§ 6. V. Mill thus states the problem to be solved:—

"We have, then, to inquire, on the present occasion, not into judgment, but judgments; not into the act of believing, but into the thing believed. What is the immediate object of belief in a proposition? What is the matter of fact signified by it? What is it to which, when I assert the proposition, I give my assent, and I call upon others to give theirs? What is that which is expressed by the form of discourse called a proposition, and the conformity of which to fact constitutes the truth of the proposition[1]?"

§ 7. Mill declares at the outset that a proposition is not about our ideas or concepts of things, but about things themselves, and dismisses all the theories of predication which have our ideas or concepts for the subject and the predicate of the proposition, with the remark that "the notion that what is of primary importance to the logician in a proposition is the relation between the two *ideas* corresponding to the subject and predicate (instead of the relation between the two phenomena which they respectively express) seems to me one of the most fatal errors ever introduced into the philosophy of Logic, and the principal cause why the theory of the science has made such inconsiderable progress during the last two centuries[2]." He then points out that Hobbes's theory that a predicate is a name of that of which the subject is a name, is a sufficient account when 'A' and 'B' are both proper names, but that it is inadequate for all propositions whose subject and predicate are not proper names, because it entirely overlooks the meaning of names in connotation.

[1] Mill's *Logic*, Vol. I. p. 99. [2] *Ibid.* p. 98.

§ 8. Mill then shows that the Denotative or Class Theory of Predication accordingly to which predication consists in referring something to a class, *i.e.*, in placing an individual under a class or one class under another, is hardly better than the theory of Hobbes. "There is," says he, "no real difference, except in language, between this theory of predication and the theory of Hobbes. For a class is absolutely nothing but an indefinite number of individuals denoted by a general name. The name given to them in common is what makes them a class. To refer anything to a class, therefore, is to look upon it as one of the things which are called by that common name. To exclude it from a class, is to say that the common name is not applicable to it[1]." The Class Theory of Predication is, argues Mill, moreover psychologically false. For in the proposition 'snow is white,' I am not thinking of 'white objects' as a class, but only of 'snow' as an object and the sensation of 'white' which it gives me.

§ 9. A view that is closely connected with the Denotative or Class Theory of Predication, and is, in fact, only a special development of it, is the equational view of propositions. According to this view, the proposition 'A is B' is an equation, 'A' and 'B' corresponding to the two sides of the equation, and 'is' to the sign of equality between them; and the meaning of the proposition is that the things denoted by 'A' are identical with those denoted by 'B.' This view is adopted by Hamilton in his later writings. It is the direct consequence of the doctrine of the Quantification of the Predicate. This doctrine is, that in thought the quantity of the predicate as well as that of the subject is implicitly contained, and that, according to the principle, that "Logic postulates to be allowed to state explicitly in language all that is implicitly contained in the thought," it may be expressed by such words as 'some,' 'all,' &c., before the predicate.

Adopting this doctrine, Hamilton obtains the following eight

[1] Mill's *Logic*, Vol. I. p. 104.

forms of propositions instead of the four we have given in a previous chapter:—

(1) All A is some B. (A.)
(2) All A is all B. (U.)
(3) No A is any B. (E.)
(4) No A is some B. (η.)
(5) Some A is some B. (I.)
(6) Some A is all B. (Y.)
(7) Some A is not any B. (O.)
(8) Some A is not some B. (ω.)

Mill objects to the adoption of the above view on the following grounds[1]:—(1) The theory is psychologically false, because the predicate of a proposition is not thought of in its extension, but only in its comprehension. In the proposition "all oxen ruminate," nobody thinks of other ruminating animals, and none ever asks the question whether or not there are other animals that ruminate; all that anyone is thinking of is the phenomenon or attribute of ruminating in reference to 'oxen.' (2) All reasoning being carried on in the ordinary forms of expression, it is desirable that every proposition in logical form should be the exact equivalent of some proposition in the common form. On this ground the proposition "all A is all B" is inadmissible, because there are none corresponding to it in ordinary language, because it is really a compound of two ordinary propositions, *viz.*, "all A is B" and "all B is A"; since it can never be accepted without proving these two. Similarly, if you take "some A is B" to mean "some A is some B only," you not only change the real logical meaning of 'some' as meaning 'not none,' it may be 'all,' into 'a part only,' 'not the whole,' but you make the proposition "some A is some B" really a double judgment, an implicit expression of the two explicit judgments, *viz.*, "some A is some B" and "some other A is not any B." (3) Logic should start with the simplest or most elementary judgments. But "all A is all B," "some A is

[1] Mill's *Examination of Hamilton's Philosophy*, Chap. XXII.

some B" are complex, consisting of two as we have just seen, while "A is B" is the simplest and most elementary, than which there cannot be any simpler.

Hamilton anticipates some of Mill's objections. He says:—But, in fact, ordinary language quantifies the predicate so often as this determination becomes of the smallest import. This it does either directly, by adding *all*, *some*, or their equivalent predesignations, to the predicate; or it accomplishes the same end indirectly, in an exceptive or limitative form. (*a*) Directly,—as "Peter, John, James, &c., are *all* the Apostles," "Mercury, Venus, &c., are *all* the planets." (*b*) But this is more frequently accomplished indirectly, by the equipollent forms of limitation or inclusion, and exception. For example, by the limitative designations, *alone* or *only*, we say, "God alone is good," which is equivalent to saying, God is all good, that is, God is all that is good; "Virtue is the only nobility," that is, virtue is all noble, that is, all that is noble. "Faith, hope, charity, alone justify." "Of animals man alone is rational," that is, man is all rational animal. "What is rational is alone or only risible," that is, "all rational is all risible, &c." Of the exceptive form Hamilton gives the following examples:—"On earth there is nothing great but man," which means "Man is all earthly great." "In man there is nothing great but mind," which means "Mind is all humanly great," that is, "all that is great in man[1]."

[1] The following note by Hamilton on the import of what are called exclusive and exceptive particles is worth quoting:—They are, "one, only, alone, exclusively, precisely, just, sole, solely; nothing but—not—except, beyond. (1) These particles annexed to the subject predesignate the predicate universally, or to its whole extent, denying its particularity or indefinitude, and definitely limiting it to the subject alone; as, 'man alone philosophises,' 'the dog alone barks,' 'man only is rational,' 'of material things there is nothing living (but) not organized, and nothing organized not living,' 'God alone is to be worshipped,' 'some men only are elect.' (2) Annexed to the predicate, they limit the subject to the predicate, but do not define its quantity, or exclude it from other subjects; as, 'Peter only plays,' 'the sacra-

"The non-quantification of the predicate in thought," argues Hamilton, "is given up by the logicians themselves, but only in certain cases where they were forced to admit, and to the amount which they could not possibly deny. The predicate, they confess, is quantified by particularity in affirmative, by universality in negative, propositions. But why the quantification, formal quantification, should be thus restricted in thought, they furnish us with no valid reason[1]."

§ 10. Mill's own theory, which may be called the Connotative or Attributive Theory of Predication, is that the proposition 'A is B' expresses a certain relation between the attributes connoted by 'A' and 'B' respectively,—or, more properly, a certain connection or relation between the phenomena on which the attributes are respectively founded and through which they are known,—and that the relation expressed by it is that of co-existence, succession, causation, resemblance, or mere existence[2]. Take, for example, the proposition "All men are mortal":

ments are only two,' 'the categories are only ten,' 'John drinks only water.' (3) Sometimes the particles sole, solely, single, alone, only, &c., are annexed to the predicate as a predesignation tantamount to *'all'*; as, 'God is the single,—one,—alone,—only,—exclusive,—adequate,—object of worship.'"

[1] Hamilton's *Lectures*, Vol. IV. pp. 261—5.

[2] In the case of a proposition whose subject is a proper name and has, therefore, according to Mill, no signification in connotation, the meaning of the proposition, according to him, is, that the attribute or attributes connoted by the predicate belong to the individual thing denoted by the subject. For example, the proposition "Socrates is a philosopher" means that the attributes of being a philosopher belong to the individual denoted by the proper name Socrates. If both the subject and the predicate of a proposition are proper names, then, according to Mill, Hobbes's theory is a sufficient account of it: as examples of such propositions he gives:—'Tully is Cicero,' 'Hyde was Clarendon,' &c., the whole meaning of such propositions is, that the predicate is a name or meaningless mark for the same thing for which the subject is a mark.

its meaning is that the objects denoted by the subject possess the attributes connoted by the predicate. The objects are not, however, individually designated. "They are pointed out only by some of their attributes; they are the objects called '*men*,' that is possessing the attributes connoted by the term 'man,' and the only thing known of them may be these attributes; indeed the proposition is general, and the objects denoted by the subject are, therefore, indefinite in number, most of them are not known individually at all. The assertion is, * * therefore, that the attributes which the predicate connotes are possessed by each and every individual possessing certain other attributes, that whatever has the attributes connoted by the subject has also those connoted by the predicate, that the latter set of attributes constantly accompanies the former set. Whatever has the attributes of man has the attribute of mortality; mortality constantly accompanies the attributes of man[1]."

To the objection that we naturally construe the subject of a proposition in its extension, and the predicate in its intention, Mill replies that "though it is true that we naturally construe the subject of a proposition in its extension, this extension, or, in other words, the extent of the class denoted by the name is not apprehended or indicated directly, and that it is both apprehended and indicated solely through the attributes."

But what is an attribute? "Every attribute," says Mr Mill, "is grounded on some fact or phenomenon, either of outward sense or of inward consciousness; and to possess an attribute is another phrase for being the cause of, or forming part of, the fact or phenomenon upon which the attribute is grounded[2]." The proposition 'All men are mortal,' therefore, really means that "wherever the various physical and mental phenomena on which the attributes of 'man' are grounded are all found, there we have assurance that the other physical and mental phenomenon, called death, will not fail to take place. The proposition does not affirm *when;* for the connotation of the word 'mortal' goes

[1] Mill's *Logic*, Vol. I. p. 109. [2] *Ibid.* p. 109.

no farther than to the occurrence of the phenomenon at some time or other, leaving the particular time undecided[1]." The relation asserted here between the two sets of phenomena is one of either co-existence or succession. Similarly in the propositions 'A generous person is worthy of honor,' 'Thoughtlessness is dangerous,' 'Prudence is a virtue,' the relation expressed is co-existence or succession, and the things between which the relation exists are the attributes connoted or signified by the subject and the predicate of the proposition, or rather the phenomena and actions upon which they are grounded.

Besides co-existence and sequence propositions may express causation or mere existence, as in the case of nöumena, or resemblance, as in such propositions as this, 'The heat of to-day is equal to the heat of yesterday.' These relations are expressed not only between phenomena, but also between nöumena, and between phenomena and nöumena. The relation of causation is only provisionally recognized, subject to the analysis of it under the head of causation.

Mill thus sums up the result of his investigation:—

"Existence, co-existence, sequence, causation, resemblance, one or other of these is asserted or denied in every proposition which is not merely verbal. This five-fold classification is an exhaustive classification of matters of fact, of all things that can be believed or tendered for belief; of all questions that can be propounded and all answers that can be returned to them[2]." On the suggestion of Professor Bain that co-existence is of two kinds,—one in different places at the same time, and the other in the same part or place, as the co-existence or co-inherence in every atom of gold, of the attributes of a certain specific gravity, tenacity, fusibility, lustre, colour, &c., Mill divides all co-existence and succession into Order in Time and Order in Place, the former including Bain's Coinhering Attributes. Of the five classes given by Mill, Bain adopts only three:—(1) Co-existence, (2) Succession, including Causation, (3) Equality or Inequality.

[1] *Logic*, Vol. I. p. 110. [2] *Ibid.* p. 116.

§ 11. A few remarks on Mill's Theory:—

The first remark to be made on Mill's theory is, that he does not show, either deductively or inductively, either from the nature of relations or from an enumeration of them, that his five-fold classification is an exhaustive one; that every possible relation between attributes has been included in his list.

The second remark is, that Mill does not give a sufficient account of the meaning of those propositions which he calls verbal. By calling them verbal, a name not without a touch of contempt, he seems to consider them as of no importance. But they are as important as those which he calls real propositions. Kant calls the two classes analytical and synthetical, respectively, and these two terms seem to express the distinction between them much better than Mill's names. What is the meaning of a verbal proposition[1] even on Mill's own theory? It is that the connotation of the predicate is a part of the connotation of the subject, that is, the phenomena on which the attribute signified by the predicate is grounded are a part of the phenomena on which the attributes connoted by the subject are grounded. The meaning of the proposition 'Man is rational,' for example, is that the phenomena on which the attribute, rationality, is grounded are a part of, or included in, the phenomena on which the attributes signified by the term 'man' are grounded. Thus it would seem, that, to the five heads given by Mill, a sixth, namely, inclusion or containing of attributes, should be added. This last is different from any that are mentioned by Mill. It is not the same as co-existence, for two phenomena or attributes may co-exist without one forming a part of the other. Thus gravity and inertia co-exist, but one is not contained in the other; while animality is contained in humanity. A verbal proposition does not merely explain the meaning of a name, but expresses, like a real proposition, a relation between phenomena or attributes. The relation expressed by it is that of containing or inclusion. The different relations between phenomena or attributes may be thus shown in a tabular view:—

[1] See Appendix F.

RELATIONS,

- Expressed by Analytical propositions.
- Expressed by Synthetical propositions.
  - Co-existence, Sequence, Causation, Existence, Resemblance.

When both the subject and the predicate of a proposition are taken in intension, Hamilton seems to recognize only one relation between them, namely, the relation of containing or inclusion; and this he does by enlarging the intension of the subject, that is, by putting into it all that is known of the thing or things denoted by the subject. Thus, according to him, all judgments are analytical or verbal, the attribute signified by the predicate being a part of the intension of the subject. He says in the *Lectures*, Vol. II. p. 336, quoting with approval from Crousary:—

"Every time we judge, we compare a total conception with a partial, and we recognize that the latter really constitutes a part of the former." Again, "when we judge, we must have, in the first place, at least two notions; in the second place we compare these; in the third, we recognize that the one contains or excludes the other; and in the fourth, we acquiesce in the recognition." "When I say, 'body is divisible,' among the notions which occur in forming my conception of body, I particularly attend to that of 'divisible,' and finding that it really agrees with the others, I judge accordingly that the body is divisible."

Another remark suggested by Mill's theory is, that it makes the meaning of a proposition depend upon what is more or less variable, indefinite, and uncertain. Take, for example, the proposition 'Man is mortal.' According to Mill its meaning is that 'mortality' co-exists with 'humanity,' that whatever has the attribute 'humanity' has the attribute 'mortality.' Now, what is meant by 'humanity'? What are the essential elements of it? Is it possible to give a final definition of it? If not, how am I to know what does and what does not possess it? Again,

the connotation of the term 'man' is not the same to all persons, being different to different classes according to the kind and degree of their education and experience. Nor is it anything constant and fixed. On the contrary, it must vary with the progress in our knowledge of man in all his aspects. Or take the proposition 'All material bodies gravitate.' Its meaning, according to Mill, is that whatever has the attribute of a 'material body' has also the attribute of 'gravitating.' Now, what are the attributes of a material body? How am I to know whether a particular body is material or not? Is the luminiferous ether (the medium of light), for example, material? Thus the connotation of terms being variable and uncertain, the meaning of a proposition, on Mill's theory, must partake of its uncertainty, variability, and indefiniteness.

The last remark that I will make on Mill's theory is connected with the import or real meaning of a term, and should, perhaps, have been made first. In the chapter on Terms, Mill says that a common or general term directly signifies objects or things, and implies or indirectly signifies attributes; so the connotation of a term is taken in that chapter to be its implied or indirect meaning, and its denotation the direct or explicit meaning[1]. But in his theory of the Proposition, the former is taken as the direct or essential meaning, while the latter is entirely passed over. Consistency seems to require that Mill should have regarded the connotative or rather attributive meaning of a term as its direct and explicit meaning, and the denotative meaning as indirect and implicit.

§ 12. From what we have given above of the views of Logicians, it is evident that they differ (1) as to the relation of A and B (subject and predicate) and (2) as to the way in which A and B are to be interpreted (that is, the meaning of subject and predicate).

[1] Mill's *Logic*, Vol. I. pp. 31, 32.—"A connotative term is one which denotes a subject, and implies an attribute," p. 31. Again, "The name is, therefore, said to signify the subjects directly, the attributes indirectly, &c.," p. 32.

As regards the first point, Hamilton, for instance, recognizes the relation of containing or not-containing (inclusion or exclusion) either in the quantity of extension or in the quantity of comprehension, arising from the 'relation of congruence or confliction.' Mansel holds that the two sets of attributes expressed by A and B must be capable of existing together in some possible object of intuition, that is, the relation of A and B is that of compatibility or incompatibility. According to Ueberweg the relation of A and B must correspond to an objective relation, that is, to a relation really existing among things. Martineau recognizes the relation of substance and attribute, and, also, the relations of time and space, of cause and effect, and of resemblance and difference. Mill gives the relations expressed by all propositions under five heads: (1) Existence, (2) Co-existence, (3) Succession, (4) Causation, (5) Resemblance. Bain includes all under three classes, (1) Co-existence, (2) Succession, (3) Equality or Inequality.

The different views arising from difference on the second point, namely, the way in which A and B are interpreted by Logicians, may be noted as follows:—(1) The Ordinary or Predicative View in which A is taken in denotation (or extension) and B in connotation (or comprehension), and the relation of A and B is that of subject and attribute. "The light," says Dr Venn, "in which a proposition has to be consistently interpreted on this view is that of *predication*. We distinguish between subject and attribute here, and we assert that a given subject does or does not possess certain attributes[1]." Of the four forms A, E, I, O, arising from this view of propositions, Dr Venn says, "These forms appear to be naturally determined by the ordinary needs of mankind, and the ordinary pre-logical modes of expressing those needs; all that Logic has done being to make them somewhat more precise in their signification than they conventionally are[2]." Again, "As just remarked, these forms of proposition certainly seem to represent the most primitive and natural

[1] *Symbolic Logic*, p. 3. [2] *Ibid.* p. 3.

modes in which thought begins to express itself with accuracy[1]."

According to this view, all relations expressed by propositions may be reduced to the single type of the relation of subject and attribute. The subject of a proposition may be anything that can possess an attribute or attributes. It may be a substance, a phenomenon, or an attribute. The predicate of a proposition is an attribute; and even when the predicate is a concrete term, the term is interpreted in its connotation (or comprehension).

This view of Propositions does not ignore the relations of space and time, of cause and effect, of resemblance and difference, expressed by many propositions; but it holds that, for logical purposes, they may all be reduced to the relation of subject and attribute. Some Logicians holding this view so far as a certain class of propositions, namely, those expressing the relation of substance and attribute, are concerned, maintain that the other relations, such as those of time and space, of cause and effect, of resemblance and difference, can not, or should not, be reduced to the single type of subject and attribute. According to them, there are different classes of propositions founded upon different categories of thought and giving rise to distinct types of reasoning[2].

[1] *Symbolic Logic*, p. 4.

[2] The relation of subject and attribute is also called the relation of substance and attribute. For the purposes of this work it is not necessary to inquire into the nature of this relation, or into the meaning of Subject, Substance, Thing, or Attribute, or to discuss the question as to whether an attribute possessing attributes becomes a substance (or thing), or remains an attribute. For the Predicative view, it is sufficient if propositions expressing other relations can, in some way, be understood to express the relation of subject and attribute; and this may be done in the following manner:—The proposition "A is equal to B," for example, expressing the relation of Equality, means, according to this view, that the attribute of being equal to B is possessed by A, whether A and B be things or attributes; the proposition "A is the cause of B," expressing the relation of

(2) The Denotative View, in which both A and B are taken in denotation (or extension). This view includes (*a*) Hobbes' View, (*b*) the Class View, in which the class or group of things denoted by A is included in the class or group of things denoted by B, and (*c*) the Equational View, in which the things denoted by A are the same as those denoted by B.

(3) The Connotative or Attributive View, in which both A and B are taken in connotation, and the relation expressed by the proposition is variable and depends on the nature of A and B. Mill adopts this view, and gives, as we have seen, the fundamental relations or matters of fact expressed by real propositions under five heads:—(1) Existence, (2) Order in time, (3) Order in place, (4) Causation, and (5) Resemblance (see p. 103). But, for the purposes of Syllogistic Logic, he gives also a general expression for it. "This, then," he says, "is the theory of the Import of Propositions, reduced to its ultimate elements: but there is another and a less abstruse expression for it, which, though stopping short in an earlier stage of the analysis, is sufficiently scientific for many of the purposes for which such a general expression is required. This expression recognises the commonly received distinction between subject and attribute, and gives the following as the analysis of the meaning of propositions:—Every proposition asserts, that some given subject does or does not possess some attribute; or that some attribute is or is not (either in all or in some portion of the subjects in which it is met with) conjoined with some other attribute.[1]"

(4) The Denotative-Connotative View, in which A and B are taken both in denotation (or extension) and in connotation (or comprehension), and the relation of A and B is a twofold one. Hamilton, for instance, holds that when both A and B are taken in extension, A is contained in B, and that when both A and B are taken in comprehension, B is contained in A.

Cause and Effect, means, according to this view, that the attribute of being the cause of B is possessed by A whatever A and B may be.

[1] Mill's *Logic*, Vol. I. p. 180. See below Appendix A, Mill's *Canons*, pp. 282—284.

There is another point on which Logicians differ in their views of the Proposition. It is connected with the different views which they take of Logic as a science. The different views of the Proposition arising from difference on this point may be noted as follows:—

(1) The Conceptualist or Subjective View, in which both A and B are concepts not necessarily corresponding to really existing things, but true of possible things, that is, of things that may be realised in Thought.

(2) The Materialist or Objective View, in which both A and B are concepts corresponding to really existing things, and the relation of A and B is a relation of concepts corresponding to a relation of things: e.g. Ueberweg's view.

(3) There is another view which is usually identified with the second view, but which should be distinguished from it. I mean the view according to which A and B stand for really existing things, and the relation of A and B is a relation of things: e.g. Spencer's view.

Mill, in his *Examination of Hamilton's Philosophy*, holds the second view; but in his *System of Logic* he very nearly gives it up and passes on to the third view. Among English Logicians he seems to occupy an intermediate position between subjective or conceptualist Logicians, represented by Hamilton and Mansel, and objective Logicians, represented by Mr Spencer and Mr Carveth Read.

The difference between the second and the third view, is that, according to the former, the two terms of a proposition are two concepts corresponding to really existing things, while, according to the latter, the two terms are really existing things or phenomena themselves. The upholders of the second view treat in Logic of the forms and relations of Thought as corresponding to the forms and relations of Things, while the upholders of the third view treat of the forms and relations of things themselves[1].

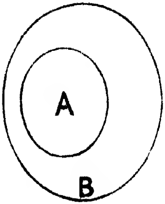
[1] See Appendix E, "The Nature and Province of Objective Logic."

CHAPTER III.

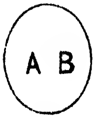
The Meaning and Representation of A, E, I, O by Diagrams.

§ 1. A stands for any Universal Affirmative proposition of the type 'All A is B.' It may be represented by the two diagrams, A, 1, and A, 2. According to the ordinary or predicative view of propositions, the meaning of A is that the attribute connoted by 'B' belongs to all the things or objects denoted by 'A,' and the implication is that it may or may not belong to any other things. The diagrams represent this, thus,—the circle A stands for the things denoted by the term A, and the circle B for the cases in which the attribute connoted by the term B occurs; the first diagram shows that these cases are more numerous than the things, and the second shows that the two are equal. The meaning of the proposition will be represented by one or other of the two diagrams.

A, 1.



A, 2.



According to the denotative view of propositions, the meaning of A is that the whole of the class denoted by the term A is included in the class denoted by the term B, or that the former is co-extensive with the latter. And this is shown by the diagrams,—in the first, the whole of the class A is a part of the class B, and in the second, the two classes coincide. The mean-

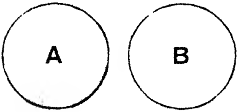
ing of the proposition will be represented by one or other of the two diagrams.

According to the connotative view of propositions, the meaning of A is that the attribute connoted by 'B' accompanies the attribute connoted by 'A' in every case, that is, wherever the latter is, there the former is. The diagrams may be understood to represent this, thus,—the first shows that the cases in which the attribute connoted by A occurs are a part of, or are less numerous than, the cases in which the attribute connoted by B occurs; the second shows that the two classes of cases coincide or are equal in number.

Thus, on all the three views, A can be represented by these two diagrams. On each of them, the subject of A is always taken in its whole extent, while the predicate is always taken in a partial and sometimes also in its total extent. This is plainly the case on the first and second views. On the third, too, this is the case, because in all cases the attribute connoted by A is accompanied by the attribute connoted by B. This fact is what is meant by saying that, *in an A proposition, the subject is distributed, and the predicate undistributed.* By the extent of an attribute is meant the number of cases in which it occurs.

§ 2. E stands for any Universal Negative proposition of the type 'No A is B.' It is represented by the following diagram. The meaning of the diagram is different on the different views of propositions.

E.



On the first view, the circle A stands for the things denoted by the term A; and the circle B for the cases in which the attribute connoted by the term B occurs; and the diagram shows that the one set is quite distinct from the other,—that the attribute connoted by B does not in any case belong to any of the things denoted by A.

On the second view, the two circles A, B stand for two classes denoted respectively by A and B; and the diagram shows that

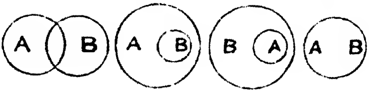
the one class is entirely excluded from the other, that the things denoted by B are quite distinct from those denoted by A.

On the third view, the circle A stands for the cases in which the attribute connoted by A occurs, and the circle B for the cases in which the attribute connoted by B occurs; and the diagram shows that the two sets do not coincide, even in a single instance.

On all the three views, then, the diagram represents the meaning of an E proposition, and shows that both A and B are taken in their entire extent, or in all cases wherever they are found. This last fact is what is meant by saying that *both the subject and the predicate of an E proposition are distributed.*

§ 3. I stands for any Particular Affirmative proposition of the form 'Some A is B.' The meaning of 'some' in logical propositions, as we have already noted, is 'not none,' 'at least one.' It does not mean a *part only.* Its universal and necessary meaning is, at least one; but it does not necessarily exclude the rest. It may mean 'many,' 'most,' 'nearly the whole,' and does not exclude 'the whole' or 'all.' In accordance with this signification of the word 'some,' the proposition 'Some A is B' is represented by the following four diagrams, each of which shows that at least one A is B.

I, 1. I, 2. I, 3. I, 4.



On the first view the meaning of I is that at least one thing, and that, it may be, every thing, denoted by A, has the attribute connoted by B; and this is represented by the diagrams thus:—each of them shows that at least one thing or a part of the things coincides with the cases, while two of them (I, 3 and I, 4) show also that the whole of A may coincide with B.

On the second view the meaning of I is that at least one thing, and that, it may be, every thing denoted by A, is included

in the class denoted by B; and this is, as in the preceding case, represented by the diagrams.

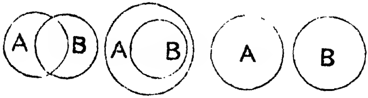
On the third view the meaning of I is that in at least one case, and that, it may be, in every case, in which the attribute connoted by A occurs, there occurs the attribute connoted by B; and this is, as in the preceding cases, represented by the diagrams.

On all the views, both the subject and the predicate are always taken in a partial extent, and sometimes also in the whole of their extent. This fact is what is meant by saying that *both the subject and the predicate of an I proposition are undistributed.*

§ 4. O stands for any Particular Negative proposition of the form 'Some A is not B.' In accordance with the logical meaning of the word 'some,' as given above, it is represented by the following three diagrams, each of which shows that at least one A is not B.

On the first view, the meaning of O is that at least one thing, and that, it may be, every thing, denoted by A, has not the

O, 1. O, 2. O, 3.



attribute connoted by 'B,'—that all the cases in which the attribute occurs are excluded from at least one thing, and, it may be, from every thing, denoted by A.

On the second view the meaning is, that at least one thing, and that it may be every thing, denoted by 'A' does not belong to the class denoted by 'B'; that the whole of the latter class is excluded from at least one, and it may be from every, individual of the former.

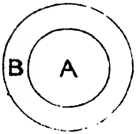
On the third view the meaning is, that in at least one case, and that it may be in every case, in which the attribute connoted by 'A' occurs, the attribute connoted by 'B' does not occur,

that every case of the latter is excluded from at least one case, and it may be from every case, of the former.

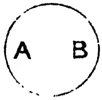
On all the views, 'B' is always taken in its entire extent, 'A' always in a part, and sometimes also in the whole of its extent. This fact is, what is meant by saying that *the predicate of an O proposition is distributed and the subject undistributed.*

§ 5. Recapitulation.—Representing 'A' and 'B,' the subject and the predicate of a proposition, by two circles, and the copula, by the mutual position or relation of the two circles, A is represented by the two diagrams (1) and (2),

(1)

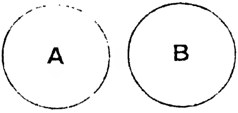


(2)



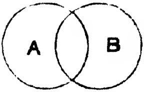
E by the single diagram (3),

(3)

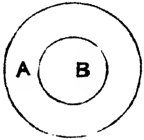


I by the four diagrams (4), (5), (6), and (7),

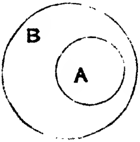
(4)



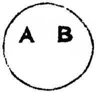
(5)



(6)

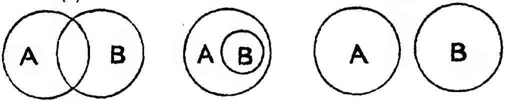


(7)



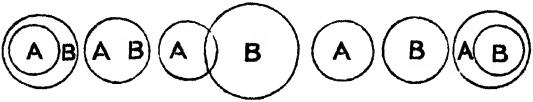
and O by the three diagrams (8), (9), and (10).

(8) (9) (10)



On a comparison of these diagrams, it will be seen that (1) and (6), (2) and (7), (3) and (10), (4) and (8), (5) and (9) are identical, and that there are altogether five fundamental diagrams. To help the memory of the student, these five diagrams are given below in a definite order:—

1st. 2nd. 3rd. 4th. 5th.



These diagrams will be henceforth called the 1st, 2nd, 3rd, 4th, and 5th respectively, and the student is advised to remember their respective numbers. A is represented by the 1st and 2nd, E by the 4th, I by the 1st, 2nd, 3rd, and 5th, and O by the 3rd, 4th, and 5th.

The subject of A is distributed, and the predicate undistributed. Both the subject and predicate of E are distributed. Both the subject and predicate of I are undistributed. The predicate of O is distributed, and the subject undistributed. That is, only universal propositions distribute their subjects, and only negative propositions distribute their predicates.

§ 6. Exercises on the meaning and representation of propositions by diagrams.

I. Show how the four propositional forms—*viz.*, A, E, I, and O —may be represented by diagrams.

II. Draw the five fundamental diagrams representing all propositions in their proper order, and state which of them represent A, which E, which I, and which O respectively.

III. Which of the four propositional forms—A, E, I, and O—may be represented by the 1st, which by the 2nd, which by the 3rd, which by the 4th, and which by the 5th diagram?

IV. Name the diagrams which represent A, E, I, and O respectively.

V. Represent each of the following propositions by its appropriate diagrams, and state its meaning according to the various theories of predication and of the import of propositions:

1. All men are rational.
2. All men are fallible.
3. Some men are rich.
4. Some elements are not metals.
5. Rain is produced by clouds.
6. Some plants have flowers.
7. All material bodies are extended.
8. No man is perfect.
9. All metals are elements.
10. All sensations are feelings.
11. Material bodies gravitate.
12. Silver is white.
13. Water boils at 100° C. under a pressure of 760 m.m.
14. Heat expands bodies.
15. Friction produces heat.

PART III.

REASONING OR INFERENCE.

CHAPTER I.

The Different Kinds of Reasoning or Inference.

A *Reasoning* is the act of the mind by which we pass from one or more given judgments to another following from them. When we pass from one judgment to another different from it, but contained in, or directly implied by it, the reasoning is called *Immediate*. When we pass from two or more judgments to another different from any of them, but justified by all of them jointly, the reasoning is called *Mediate*. The new judgment, or the judgment obtained from the given judgment or judgments, is called the *Conclusion*, and the given judgment or judgments, the *Premiss* or *Premisses*. If the conclusion be not more general than either of the premisses in a mediate reasoning, the reasoning is called *Deductive*. If the conclusion be, on the other hand, more general than any of the premisses, the reasoning is called *Inductive*. In Deductive Reasoning the conclusion is a development of what is contained in, or implied by, the premisses. In Inductive Reasoning the conclusion contains or implies more

than what is contained in or implied by any or all of the premisses. Thus we get the following kinds of reasoning:—

REASONING

- Immediate
- Mediate
  - Deductive
  - Inductive

Are there also two kinds, Deductive and Inductive, under Immediate Inference? Immediate Reasoning, as it is usually treated of, is all Deductive,—that is, in no case is the conclusion more general than the premiss. But if we define Immediate Reasoning as a reasoning in which a judgment is obtained from another judgment, it is evident, that the former may be more general as well as less general than the latter. If the conclusion be more general, the reasoning should certainly be called Inductive. If, for example, we could, in any case, draw the general conclusion from *a single instance*,—that is, from a single judgment or proposition—the reasoning, in that case, would be Immediate, as consisting of a single premiss only, and should be called Inductive, as leading to a conclusion more general than the premiss.

In Deductive Logic, however, all immediate reasoning and all mediate reasoning are deductive, and the following classification is, therefore, preferable:—

REASONING

- Deductive
  - Immediate
  - Mediate
    - Syllogistic
    - Non-Syllogistic: *e.g.*, certain mathematical deductive reasonings.
- Inductive

Reasoning is either Inductive or Deductive. The latter is again either (1) Immediate, or (2) Mediate, according as the conclusion follows from one premiss or from more than one. A Mediate Deductive Reasoning is called a Syllogism, when it conforms to the axiom called *Dictum de omni et nullo*,—"Whatever is affirmed or denied of a class distributively, may be affirmed or denied of any thing belonging to that class," or to some similar axiom or axioms. It may be called Mathematical, when it conforms to some one or other of the axioms in mathematics, such as (1) that things which are equal to the same thing are equal to one another, (2) that the sums of equals are equal, (3) the principle or axiom called *Argumentum a fortiori*, that 'a thing which is greater than a second, which is greater than a third, is greater than the third.' The subdivisions of the other main division cannot be discussed in this book.

A Reasoning, regarded *objectively*, is the inference of a relation from one or more given relations among things and attributes. When a general or universal relation is inferred from one, a few, or many particular relations, the reasoning or inference is Inductive. When the relation inferred is not more general than the given relation or relations, and is, in fact, contained in, or implied by, the latter, the reasoning or inference is called Deductive. It is Immediate when the inference is drawn from one given relation or premiss, and Mediate when drawn from more than one. The word inference, it should be noted, has, at least, three meanings:—(1) the process of reasoning, (2) the product of reasoning consisting of the premisses and the conclusion, and (3) the conclusion only. We have here used the word in the second sense, but it is frequently used in the first, and more frequently in the third.

A reasoning, expressed in language, is called an Argument. There are thus as many kinds or varieties of the latter as there are of the former. The simplest form of argument corresponding to the simplest form of reasoning, namely, Immediate, consists of two propositions,—the premiss and the conclusion. A Mediate deductive reasoning gives rise to an argument consisting of more than two propositions, namely, the premisses and the conclusion.

An Inductive reasoning gives rise to an argument consisting of many propositions, namely, the particular instances constituting the data, and the general conclusion justified by them. The word 'argument' also denotes a series of reasonings advanced to establish a certain conclusion.

It should be carefully noted that so far as Logic is concerned with reasoning, it treats of the principles of correct reasoning, and lays down the conditions to which reasoning must conform in order that it may be valid. It is no part of Logic to give an account of the various processes according to which men do or may reason, but of those according to which they ought to reason, and must reason if their reasonings are to be valid. The former is the business of the science of Psychology, the latter only is the business of Logic[1].

Examples of Different Kinds of Reasoning or Inference.

I. DEDUCTIVE.

i. Immediate.

1. All men are mortal,
   ∴ Some mortal beings are men.
2. No man is perfect,
   ∴ All men are imperfect.

ii. Mediate.

A.—Syllogistic.

3. All men are fallible,
   All prophets are men;
   ∴ All prophets are fallible.

[1] No attempt is made here to give an exhaustive account of all the processes of reasoning either from the psychological or from the logical point of view. In this chapter, the subject is treated for the purposes of this work. There is great diversity of view among Logicians (1) as to the nature of reasoning or inference,—as to what is and what is not inference, and (2) as to its fundamental kinds and varieties. The theory of Reasoning and Inference, like the theory of Predication and of the Import of Propositions, is a most important subject in Logic and Psychology, and would demand a thorough treatment in a complete treatise on Logic.

4. No man is perfect,
All philosophers are men;
∴ No philosopher is perfect.

5. All metals are elements,
Gold is a metal;
∴ Gold is an element.

B.—Non-Syllogistic.
e.g., Mathematical.

6. A is equal to B,
C is equal to B;
∴ A is equal to C.

7. A is greater than B,
B is greater than C;
∴ A is greater than C.

8. A is less than B,
B is less than C;
∴ A is less than C.

9. A is a part of B,
B is a part of C;
∴ A is a part of C.

10. A is equal to B,
C is equal to D;
∴ A + C is equal to B + D.

Mathematical reasonings are usually regarded as valid, if they conform to the axioms of mathematics. By taking the axioms as major premisses, and the data of the reasonings as minor premisses, they may, however, be reduced to the syllogistic form. Examples 6 and 7 given above may be stated syllogistically as follows:—

6. Things which are equal to the same thing are equal to one another; the two things A and C are equal to the same thing (B); therefore the two things A and C are equal to one another.

7. A thing which is greater than a second, which is greater than a third, is greater than the third; the thing A is greater than a second (B), which is greater than a third (C); therefore the thing A is greater than the third (C).

Similarly, other mathematical reasonings may be reduced to fully-expressed syllogisms.

II. INDUCTIVE.

1. Air expands by heat,
Water expands by heat,
Mercury expands by heat,
Copper expands by heat,
Gold expands by heat;
∴ All material bodies expand by heat.

2. Water is solidified by cold,
Mercury is solidified by cold,
Cocoanut oil is solidified by cold;
∴ All liquids are solidified by cold.

3. The friction of the palms of our hands against each other produces heat,
The friction of two pieces of wood produces heat,
&c., &c., &c.;
∴ The friction of all material bodies produces heat.

4. Many men whom I knew have died,
All the men in the past ages have died;
∴ All men will die.

5. The three angles of this triangle are together equal to two right angles;
∴ The three angles of any triangle are together equal to two right angles.

6. These two straight lines cannot inclose a space,
∴ No two straight lines can inclose a space.

7. An equilateral triangle can be constructed upon this finite line,
∴ An equilateral triangle can be constructed upon any finite line.

Inductive reasonings conform to the canons and rules of Induction. By taking the canons and rules as major premisses, and the data of the reasonings as minor premisses, Inductive reasonings, like mathematical, may be reduced to the syllogistic form[1].

[1] See below, Appendix D.

CHAPTER II.

OF IMMEDIATE INFERENCES.

§ 1. IMMEDIATE Inference, as a process of reasoning, is the process of deriving or deducing a proposition from a given proposition or premiss. As an argument or reasoning expressed in language, it consists of the given proposition, and the proposition necessarily following from it. As an inference or conclusion, it is the proposition thus following,—the result of the process. The derivation of a proposition from a term may also be regarded as a kind of Immediate Inference. Every attribute connoted by a term may be affirmed of the term. Thus there are two kinds of Immediate Inference.

(1) In the first kind, a proposition is inferred from a term. Take the connotative term 'man,' and let its connotation consist of the two attributes 'rationality' and 'animality.' From this term it is evident that we may at once infer the following two propositions: (i) 'Man is rational,' (ii) 'Man is animal.' This kind of immediate inference depends on the axiom that every attribute connoted by a term may be predicated of it. This axiom is the basis of the formation of verbal propositions by the analysis of the connotation of terms. This mode of immediate inference is really equivalent to the affirmation of an attribute of an aggregate of attributes, or of a thing or things, of which the attribute affirmed is known to form a part.

Exercise.

*Infer one verbal proposition from each of the following terms:—

(1) Material body.	(6) Plant.
(2) Figure.	(7) Animal.
(3) Chalk.	(8) House.
(4) Table.	(9) Man.
(5) Book.	(10) Mind.

(2) In the second kind, a proposition is inferred from a given proposition. There are seven different forms of it: *viz.*, I. Conversion; II. Æquipollence, Permutation, or Obversion; III. Contraposition; IV. Subalternation; V. Opposition; VI. Modal Consequence; VII. Change of Relation. Of these we shall treat in order.

§ 2. I.—Of Conversion.

Conversion is the admissible transposition of the subject and the predicate of a proposition. The proposition to be converted is called the *convertend*, and the proposition inferred from it the *converse*, which may be defined as a legitimate inference, having for its subject and predicate the predicate and subject, respectively, of the convertend. In an hypothetical proposition, the consequent and the antecedent are transposed. In drawing inferences by the process of conversion, the following three rules must be observed:—

(1) The subject and the predicate in the convertend must be the predicate and the subject, respectively, in the converse.

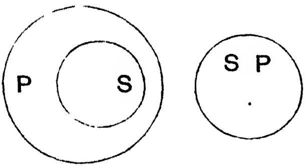
(2) No term should be distributed in the converse which was not distributed in the convertend.

(3) The quality of the converse is the same as that of the convertend,—that is, the converse of an affirmative proposition is affirmative, and the converse of a negative proposition is negative.

The first rule is evident from the definition of conversion. The second and third rules must be observed in order that the converse may be an admissible inference, that is, an inference following *necessarily* from the given proposition. The second

rule is evident from the fact that if a term is used, in the premiss, to signify *some* individuals, it can not, in the conclusion, be used to signify *every* individual, denoted by the term. The third rule follows from the meaning of an affirmative and a negative proposition. An affirmative proposition, such as S is P, means that at least one S is included in P; and from this it does not follow that at least one P is excluded from S (or P is not S), for P and S may coincide. A negative proposition, such as S is not P, means that at least one S is excluded from P; and from this it does not follow that at least one P is included in S (or P is S), for P and S may lie entirely outside of each other.

(1) From A follows I by conversion: from 'All S is P' follows by conversion 'At least one or some P is S.' This follows from the rules, and can be easily proved by the diagrams. By the third rule the converse of A must be affirmative, that is, A or I; by the second rule it can not be A; and, as no rules are violated by inferring I from A by conversion, it is I. A is represented by the first and second diagrams, and from both of these follows I, 'Some P is S.'



From the first follow I, 'Some P is S,' and O, 'Some P is not S.' From the second follow A, 'All P is S,' and I, 'Some P is S.' Thus from each of them, that is, from A in every case, follows I only by conversion.

Examples.—All men are mortal: its converse is 'Some mortal is man,' 'At least one that is mortal is man,' or 'Some mortal beings are men.' If A is, B is: its converse is 'In some cases if B is, A is.'

(2) From I follows I by conversion: from 'Some S is P,' we can infer immediately 'At least one or some P is S.' This follows from the rules, and can be easily proved by the diagrams representing I. By the third rule the converse of I must be affirmative, that is, A or I; by the second rule it can not be A; and as

no rules are violated by inferring I from I by conversion, it is I. I is represented by the 1st, 2nd, 3rd, and 5th diagrams, and from each of them it will be seen that the converse I 'Some P is S' follows. Hence the converse of I is I[1].

Examples.—Some men are wise: its converse is 'At least one wise being is man.' In some cases if A is, B is: its converse is 'In some cases if B is, A is.'

That I follows from I by conversion and that nothing else follows may be thus shown. From the 2nd and 5th diagrams representing I, follow by conversion both A and I; from the 1st and 3rd representing I, follow by conversion I and O. Thus from each of them, that is, from I in every case, follows I only by conversion.

(3) From E follows E by conversion: from 'No S is P' follows 'No P is S.' This is at once evident from the 4th diagram representing E, and follows also from the rules. By the third rule the converse of E must be negative, that is, E or O; and as no rules are violated by inferring E from E by conversion, it is E. O also follows; but it is useless to infer O where E can be inferred.

Examples.—No man is perfect: its converse is 'No perfect being is man.' If A is, B is not: its converse is 'If B is, A is not.'

(4) From O nothing follows by conversion: this follows from the rules, and can be proved by the diagrams. By the third rule the converse of O must be negative, that is, E or O; and, as the second rule is violated by inferring E or O from O by conversion, there is no converse of O.

O, 'Some S is not P,' is represented by three diagrams, *viz.*, the 3rd, 4th, and 5th.

From the 3rd follow O and I by conversion: Some P is not S, and Some P is S.

[1] The student should draw the respective diagrams in this case as well as in those that follow, and satisfy himself that the conclusions asserted to follow do really follow from them.

From the 4th follow E and O by conversion: No P is S, and Some P is not S.

From the 5th follow A and I by conversion: All P is S, and Some P is S.

Hence, from all the three forms of O, or from O in all cases, nothing follows by conversion. From the 3rd and 4th follows O; but as O does not follow from the 5th diagram, we cannot infer it from every form of O. From the 3rd and 5th follows I; but, as I does not follow from the 4th diagram, it can not be inferred from O.

Recapitulation.—The converse of I is I; and the converse of E is E. The converse in these two cases has the same quality and quantity as the convertend; and when this is the case, the process of conversion is called *Simple Conversion.* The converse of A is I. The converse, or the inferred proposition in this case, is particular, while the convertend is universal; and when this is the case, the process of conversion is called *Conversion per accidens or by limitation.* O cannot be converted.

Exercise.

Convert the following propositions:—

1. All material bodies are extended.
2. Some animals are birds.
3. No man is immortal.
4. Hydrogen is the lightest body known.
5. Benevolence is a virtue.
6. Every element is not a metal.
7. Certain metals are ductile.
8. Some animals have no power of locomotion.
9. Matter is indestructible.
10. None but elements are metals.
11. If mercury is heated, it expands.
12. If a judgment is analytical, it is not synthetical.
13. If a judgment is not synthetical, it is analytical.
14. In some cases a sensation is followed by a perception.
15. In some cases a sensation is not followed by a perception.
16. Only a man of genius can hope for success without industry.

17. All upright men are not indifferent to flattery.

*18. There are few students who have a taste both for physics and metaphysics.

19. No one can hope for success without industry.

20. Knowledge is power.

§ 3. II.—Obversion, Permutation, or Æquipollence.

This process of immediate inference consists in taking the contradictory of the predicate of the given proposition as the predicate of the inference, and then changing the quality of the proposition. The inference, or the proposition inferred, is called the *Obverse* or *Permutation*, and the given proposition may be called the *Obvertend*. The obverse of a proposition may be defined as an admissible inference, having for its subject and predicate the subject and contradictory of the predicate, respectively, of the proposition.

(1) From A follows E by obversion: from the proposition 'All S is P' follows the proposition 'No S is not-P.' This is evident from the two diagrams, 1st and 2nd, representing A, from both of which follows the proposition 'No S is not-P,' 'No S is other than P.'

Example.—All men are mortal: its obverse is 'No men are not-mortal.'

(2) From E follows A by obversion: from 'No S is P' follows 'All S is not-P,' *i.e.*, every S lies in the region of not-P, or outside P. This is evident from the 4th diagram representing E.

Example.—No men are perfect: its obverse is 'All men are not-perfect.'

(3) From I follows O by obversion: from 'Some S is P' follows 'Some S is not not-P,' or 'Some S is not other than P.'

This can be proved from the diagrams: I is represented by the 1st, 2nd, 3rd, and 5th diagrams, from each of which follows the proposition 'Some S is not not-P,' *i.e.*, some S is excluded from the whole of the region belonging to not-P.

Example.—Some men are wise: its obverse is 'Some men are not not-wise.'

(4) From O follows I by obversion: from 'Some S is not P' follows 'Some S is not-P.' O is represented by the 3rd, 4th, and 5th diagrams, from each of which follows the proposition 'Some S is not-P,' or some S lies in the region of not-P.

Example.—Some elements are not metals: its obverse is 'Some elements are non-metals.'

An hypothetical proposition may be obverted by taking the contradictory of the consequent as the consequent in the inference and then changing the quality of the given proposition. The antecedent and the consequent of an hypothetical proposition correspond, respectively, to the subject and the predicate of a categorical proposition; and the quality of an hypothetical proposition is determined by the quality of its consequent. In a previous chapter (see pp. 68, 69), it has been shown that the antecedent and the consequent of an hypothetical proposition, though they appear to be assertions, consist really of two many-worded terms. In the proposition, "If A is B, C is D," the antecedent and the consequent are 'A being B' or 'AB' and 'C being D' or 'CD,' respectively. The typical hypothetical proposition is "If A, then B," whatever A and B may be. In the affirmative form, it means that B depends upon A—that there is connexion between A and B; and in the negative form, it means that B does not depend upon A—that there is no connexion between A and B. Thus the proposition "If A is B, C is D" is equivalent to "If AB is, CD is," and means that CD depends upon AB. The proposition "If A is B, C is not D" is equivalent to "If AB is, CD is not," and means that CD does not depend upon AB—that there is no connexion between AB and CD.

Examples.

1. "If A is, B is": its obverse is "If A is, not-B is not."

2. "If A is B, A is C": its obverse is "If A is B, A is not not-C," which is equivalent to "If AB is, not-CA is not."

3. "If A is B, C is D": its obverse is "If A is B, C is not not-D," which is equivalent to "If AB is, not-DC is not."

4. "If A is, B is not": its obverse is "If A is, not-B is."

5. "If A is B, C is not D": its obverse is "If A is B, C is not-D," which is equivalent to "If AB is, not-DC is."

6. "If a triangle is equilateral, it is equiangular": its obverse is "If a triangle is equilateral, it is not non-equiangular," which is equivalent to "If an equilateral triangle is, a non-equiangular triangle is not," i.e. the non-equiangularity of a triangle does not depend upon, or coexist with, the attribute of its being equilateral.

7. "If it rains, the ground will be wet": its obverse is "If it rains, the ground will not be not-wet," which is equivalent to "If raining is, not-wet ground is not," i.e. the ground being not-wet does not depend upon raining.

Exercise.

Obvert the following propositions:

1. All sensations are feelings.
2. Every phenomenon has a cause.
3. Only material bodies gravitate.
4. Some plants have no flowers.
5. Justice is a virtue.
6. If A is B, A is not C.
7. If A is not B, C is not D.
8. If A is not B, C is D.
9. If a term is singular, it is not general.
10. If a body is heated, it rises in temperature.
11. If there is a chemical action, there is an evolution of heat.
12. If all impeding causes were removed, a body once in motion would continue to move for ever.

§ 4. III.—Contraposition.

Contraposition consists in taking the contradictory of the predicate of the given proposition as the subject of the inference, and the subject as the predicate, and then changing the quality or both the quality and the quantity of the proposition, if required. The inference, or the proposition obtained by contraposition, is called the *Contrapositive.* The contrapositive of a proposition may be defined as an admissible inference, having for its subject and predicate the contradictory of the predicate and the subject, respectively, of the proposition.

(1) From A follows E by contraposition: from 'Every S is P' follows 'No not-P is S.' Here 'not-P,' the contradictory of the predicate of the given proposition (Every S is P), is taken as the subject of the inference, and the quality is changed from affirmative to negative.

This is evident from the diagrams, 1st and 2nd, representing A, from each of which follows the proposition 'No not-P is S,' *i.e.*, all S is excluded from the region of Not-P.

Example.—All men are mortal: its contrapositive is 'No not-mortal is man.'

(2) From E follows I by contraposition: from 'No S is P' follows 'Some not-P is S.' This is evident from the 4th diagram representing E. In this case the quantity of the contrapositive is particular, while the given proposition is universal.

Example.—No man is perfect: its contrapositive is 'Some not-perfect is man.'

(3) From O follows I by contraposition: from 'Some S is not P' follows 'Some not-P is S.' This may be proved from the diagrams, 3rd, 4th and 5th, representing O:—

From the 3rd follows I by contraposition: Some not-P is S. From the 4th and 5th also follows I. Hence from each of the three forms, or from O in every case, follows I by contraposition.

Example.—Some elements are not metals: its contrapositive is 'Some non-metals are elements.'

(4) From I follows no conclusion by contraposition. This may be proved thus:—

I is represented by the 1st, 2nd, 3rd, and 5th diagrams.

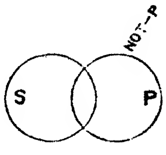
From the 3rd and also from the 5th follows by contraposition I, Some not-P is S. But from the 1st and 2nd, I does not follow. Hence from all the forms of I, that is, from I in every case, I (Some not-P is S) cannot be inferred by contraposition.

Again, from the 1st, 2nd and 5th follows O (Some not-P is not S); but it does not follow from the 3rd diagram, and therefore O (Some not-P is not S) cannot be inferred from all the forms of I.[1]

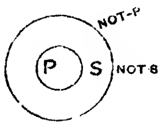
Two diagrams (3rd and 5th) allow I, and two others (1st and 2nd) allow O; but from *each* of them neither I nor O can be inferred. Hence I cannot be contraposed.

Recapitulation.—The contrapositive of A is E, of E I, and of O I, while I cannot be contraposed. The student should care-

[1] In the 3rd diagram, a part of P coincides with a part of S, and some not-P, which lies outside P and consequently outside the coinciding part of P, lies outside the coinciding part of S and not outside the whole of S,—that is, all that is known certainly is that some not-P is excluded from a part, and not from the whole, of S; or, in other words, the proposition "Some not-P is not S" is not true. From the 5th diagram follows the proposition "Some not-P is not S," on the assumption that every term has a contradictory. In Contraposition and Obversion, it is assumed that there is a term contradictory to the predicate of the premiss. A and I can not be obverted and A can not be contraposed unless their predicates have contradictory terms. See Appendix G.—Note on Obversion and Contraposition."



3rd.



5th.

fully note that I cannot be contraposed, and that O cannot be converted.

An hypothetical proposition may be contraposed by taking the antecedent and the contradictory of the consequent in the proposition as the consequent and the antecedent respectively in the inference, and then changing the quality in the case of A and O, and also the quantity in the case of E.

(1) If A is, B is: its contrapositive is 'If B is not, A never is,' 'Wherever B is not, A never is.'

(2) If A is, B is not: its contrapositive is 'In some cases if B is not, A is.'

(3) In some cases if A is, B is not: its contrapositive is 'In some cases if B is not, A is.'

NOTE.—Contraposition is also called Conversion by Negation. The older logicians converted O by this process. We have seen that the process is applicable also to A and E, and inapplicable to I only. The contrapositive of a given proposition may be regarded as the *converted obverse* of it; and contraposition as consisting in obversion and in conversion of the obverse. Some logicians have indeed regarded the inference as double and the process as two-fold, including obversion and conversion, and have accordingly excluded contraposition from Immediate inference. But we have seen that, with the aid of the diagrams, the contrapositive of a proposition can be inferred as immediately as its obverse or its converse. In contraposing a proposition according to the older method, first obvert it, and then take the converse of the obverse.

Examples.

(1) All S is P.

Its obverse is 'No S is not-P'; the converse of this obverse is 'No not-P is S,' and this last is the contrapositive of the given proposition (All S is P).

(2) No S is P.

Its obverse is 'All S is not P'; the converse of this obverse is 'Some not-P is S,' which is the contrapositive of the given proposition (No S is P).

(3) Some S is not P.

Its obverse is 'Some S is not-P'; the converse of this obverse is 'Some not-P is S,' and this last is the contrapositive of the given proposition (Some S is not P).

(4) Some S is P.

Its obverse is 'Some S is not not-P,' which is O, and O cannot be converted as we have seen before (*vide* pp. 127—8).

Exercise.

Contrapose the following propositions:—

1. All animals are mortal.
2. No created being is perfect.
3. All gases can be liquefied.
4. Some plants are not devoid of the power of locomotion.
5. Some animals are insentient.
6. Some substances have no cause.
7. All bodies that have inertia have weight.
8. If mercury is heated, it expands.
9. In some cases if a body is heated, its temperature does not rise.
10. In some cases a sensation is followed by a perception.
11. If A is B, C is D.
12. If A is B, C is not D.
13. In some cases if A is B, C is not D.
14. In some cases if A is B, C is D.
15. In all cases if A is not B, C is D.
16. In all cases if A is not B, C is not D.
17. In some cases if A is not B, C is D.
18. In some cases if A is not B, C is not D.

§ 5. IV.—Of Subalternation.

This process of immediate inference consists in passing from the universal to the particular, and from the particular to the universal, with the same subject and predicate, and of the same quality. By subalternation follows :—

(1) From the truth of A, the truth of I, and from the truth of E, the truth of O; but not conversely from the latter the former. Thus, if 'All S is P' be true, 'Some S is P' will also be

true; but if the latter be true, the former will not necessarily be true.

(2) From the falsity of I, the falsity of A, and from the falsity of O, the falsity of E; but not conversely the former from the latter. If 'Some S is P' be false, then 'All S is P' must also be false; if 'Some S is not P' be false, then 'No S is P' must be false; but not conversely, that is, the falsity of the particular does not follow from the falsity of the corresponding universal. 'All S is P' may be false, and still 'Some S is P' may be true. Similarly, E may be false, and the corresponding O true.

The proof lies in the fact (1) that I or O simply repeats what is already recognized as true by A or E, and (2) that what fails even in one case can not be universally true, or what holds good even in one case can not be universally denied. The proof of the converse lies in the fact (1) that something may be true or false in some cases, in at least one case, though not universally, and (2) that what is not true or false in all cases, may yet be true or false in *some* cases, in *at least one* case. The rules of inference given above may be easily proved also from the diagrams.

§ 6. V.—Of Opposition.

In a previous chapter (*vide* p. 78) we have seen that A and O, and E and I, are called, in relation to each other, *Contradictory Opposites*, that A and E are called, in relation to each other, *Contrary Opposites*, and that I and O are called *Subcontrary Opposites*. In consequence of the opposition which exists among A, E, I, and O, having the same subject and predicate, but differing in quality, or in both quality and quantity, when any one is given as true or false, the others are necessarily either true, false, or unknown. We shall now inquire into these necessary connections among them, and lay down certain general rules of immediate inference by opposition:—

(1) Given the truth of A (All S is P). From the truth of A, as illustrated by the 1st and 2nd diagrams, it follows that E is false and also that O (Some S is not P) is false.

(2) Given the falsity of A (All S is P). From the falsity of A, as represented by one or other of the 3rd, 4th, and 5th diagrams[1], follow the truth of O (Some S is not P); and also the truth in one case (4th), and the falsity in the other cases (3rd and 5th) of E, or, in other words, the doubtfulness or uncertainty of E (No S is P).

(3) Given the truth of E (No S is P). From the 4th diagram representing E, follows at once the falsity of A, and also the falsity of I (Some S is P).

(4) Given the falsity of E (No S is P). The falsity of E is represented by one or other of the 1st, 2nd, 3rd, and 5th diagrams, from which follow the truth of I, and also the truth of A in two cases (1st and 2nd), and the falsity of A in two others (3rd and 5th), or, in other words, the doubtfulness of A.

(5) Given the truth of I (Some S is P). From the 1st, 2nd, 3rd, and 5th diagrams representing I, follow at once the falsity of E and also the truth of O (Some S is not P) in two cases (3rd and 5th), and the falsity of O in the other two (1st and 2nd), or, in other words, the doubtfulness of O.

(6) Given the falsity of I (Some S is P). This is represented by the 4th diagram, from which follows at once the truth of E (No S is P), and also the truth of O.

(7) Given the truth of O (Some S is not P). This is represented by the 3rd, 4th, and 5th diagrams, from which follows at once the falsity of A, and also the doubtfulness of I.

(8) Given the falsity of O (some S is not P). This is represented by one or other of the 1st and 2nd diagrams, from which follows at once the truth of A (All S is P), and also the truth of I.

[1] The falsity of A means that the relation between the subject and the predicate can not be represented by the 1st and 2nd diagrams, and that it must be represented by one or other of the remaining three diagrams. The falsity of E, I, or O may similarly be represented by diagrams.

The results we have obtained above may be thus tabulated:—

	Given.	A All S is P.	E No S is P.	I Some S is P.	O Some S is not P.
1	A true.		False.	True by Subalternation.	False.
2	A false.		Doubtful.	Doubtful by Subalternation.	True.
3	E true.	False.		False.	True by Subalternation.
4	E false.	Doubtful.		True.	Doubtful by Subalternation.
5	I true.	Doubtful by Subalternation.	False.		Doubtful.
6	I false.	False by Subalternation.	True.		True.
7	O true.	False.	Doubtful by Subalternation.	Doubtful.	
8	O false.	True.	False by Subalternation.	True.	

A comparison of the results tabulated above leads to the following conclusions and rules of immediate inference :—

(1)	The falsity of	O	follows from the truth of	A.
	,,	I	,,	E.
	,,	E	,,	I.
	,,	A	,,	O.
	The truth of	O	follows from the falsity of	A.
	,,	I	,,	E.
	,,	E	,,	I.
	,,	A	,,	O.

That is, from the falsity of a proposition follows the truth of its contradictory opposite, and from the truth of a proposition follows the falsity of its contradictory opposite. Hence the rule :—*Of two propositions related to each other as contradictory opposites, one must be true and the other false.*

(2) From the truth of A follows the falsity of E, and from the truth of E, the falsity of A ; but not conversely. That is, from the truth of a proposition follows the falsity of its contrary opposite, but not conversely from the falsity of one the truth of the other. Hence the rule :—*Of two propositions related to each other as contrary opposites, both cannot be true ; one must be false, and both may be false.*

(3) From the falsity of I follows the truth of O, and from the falsity of O follows the truth of I, but not conversely, from the truth of the one the falsity of the other. Hence the rule :—*Of two propositions related to each other as subcontrary opposites, both cannot be false ; one must be true, and both may be true.*

These rules can also be shown to be true by a consideration of the propositions themselves and by particular examples. If the proposition 'All S is P' be true, *i.e.*, if 'P' can be affirmed of every 'S,' then it can not be denied of all 'S,' nor of any one 'S,' or, in other words, both E and O must be false. Similarly, if the proposition 'No S is P' be true, *i.e.*, if 'P' can be denied of every 'S,' then it can not be affirmed of a single 'S,' or, in other words, both I and A must be false. If the proposition

'Some S is P' be true, *i.e.*, if 'P' can be affirmed of at least one 'S,' then it can not be denied of *every* 'S,' and it may or may not be denied of *some* 'S,' or, in other words, E (No S is P) must be false, and O (Some S is P) true or false, *i.e.*, doubtful. If the proposition 'Some S is not P' be true, *i.e.*, if 'P' can be denied of at least one 'S,' then it can not be affirmed universally of 'S,' and may or may not be affirmed of some 'S,' or, in other words, A must be false and I doubtful. The other cases may also be similarly proved; and the results are the same as we have given above. We shall now give some concrete examples: If 'All metals are elements' be true, then its contrary 'No metals are elements' is evidently false; and its contradictory O 'Some metals are not elements' is also false; because, in the original proposition 'elements' is affirmed of 'all metals,' and therefore it can not be denied of some. The principle of consistency requires that what is affirmed of all members of a class, must not be denied of any of them. If 'Some elements are metals' be true, then its contradictory E 'No elements are metals' must be false, and its subcontrary O 'Some elements are not metals' may or may not be true.

Exercise.

Draw the inferences which follow by subalternation and opposition from the truth of the following propositions:—

1. All material bodies are extended.
2. The virtuous are rewarded.
3. No knowledge is useless.
4. Benevolence is a virtue.
5. Few know both physics and metaphysics.
6. Every phenomenon has a cause.
7. Some substances are uncaused.
8. Some books are not useful.
9. None but elements are metals.
10. All metals except one are solid.

§ 7. VI.—Modal Consequence.

By this process an inference is drawn from a given proposition by changing its modality:—

(1) From a necessary proposition follows the corresponding assertory, or problematic proposition, but not conversely from the latter the former: from 'S must be P' can be inferred 'S is P,' or 'S may be P'; but from 'S may be P' or 'S is P, we can not infer 'S must be P.' This is evident from the fact that from a higher degree of certainty, a lower can be inferred, but not from the latter the former.

(2) From the inadmissibility of a problematic proposition follows the inadmissibility of the corresponding assertory and necessary, from the inadmissibility of an assertory proposition follows the inadmissibility of the corresponding necessary; but not conversely from the latter the former. This is evident from the fact that where a lower degree of certainty is wanting, a higher degree can not be inferred, and that where a higher degree may be wanting, a lower degree may be established. If 'S may be P' be inadmissible, then 'S is P' and 'S must be P' must also be inadmissible. But the latter may be inadmissible, and still the former may be admissible. 'All men are wise' may be inadmissible, and still the proposition 'All men may be wise' may be admissible. 'He dies' may be inadmissible, and still 'He may die' may be admissible.

§ 8. VII.—Of Change of Relation.

This mode of immediate inference consists in inferring a proposition from a given proposition by changing the relation of the latter, that is, in inferring (1) a hypothetical from a categorical, (2) a categorical from a hypothetical, (3) hypotheticals from a disjunctive, (4) a disjunctive from hypotheticals.

(1) From the categorical 'All S is P' follows the hypothetical 'If S is, P is' (A).

From the categorical 'Some S is P' follows 'In some cases if S is, P is' (I).

From 'No S is P' follows 'In all cases if S is, P never is' (E).

From 'Some S is not P' follows 'In some cases if S is, P is not' (O).

(2) From the hypothetical 'If S is, P is' follows the cate-

gorical 'Every case of the existence of S is a case of the existence of P' (A).

From 'If A is B, C is D' follows 'Every case of A being B, is a case of C being D' (A).

From the proposition 'If S is, P is not' follows 'No case of the existence of S is a case of the existence of P.'

Similarly in the case of I and O.

(3) From the disjunctive 'A is either B or C' follows, according to Mill one or the other of the two following hypotheticals :—

(1) If A is not C, A is B.
(2) If A is not B, A is C.

According to Ueberweg, two more forms may be inferred :—

(3) If A is C, A is not B.
(4) If A is B, A is not C.

The rule of inference, according to Ueberweg, is, that the truth of one alternative implies the falsity of the other, and the falsity of the one the truth of the other. According to Mill, the rule is that the falsity of the one implies the truth of the other member, but *not conversely;* and that both the members *may be* true. According to Ueberweg, therefore, the two members of a disjunctive proposition are like two contradictory propositions, which can not both be true, the truth or the falsity of the one implying, respectively, the falsity or the truth of the other; while, according to Mill, they are like two subcontrary propositions, which may both be true, the falsity of the one implying the truth of the other.

From the disjunctive propositions, "This metal is either a conductor of heat or a conductor of electricity," "He who prefers a lower pleasure in presence of a higher is either immoral or imprudent," "Some men are either prophets or philosophers," may be inferred two hypothetical propositions, as according to Mill, while, from the disjunctive propositions, "This animal is either a vertebrate or an invertebrate," "The soul is either mortal or immortal," "Every organism is either a plant or an

animal," may be inferred four hypothetical propositions, as according to Ueberweg.

(4) From the four or the two hypotheticals may again be inferred the original disjunctive as follows:—

(*a*) The four hypotheticals are:—

(1) If A is not C, A is B.
(2) If A is not B, A is C.
(3) If A is C, A is not B.
(4) If A is B, A is not C.

From (4) if the proposition 'A is B' be true, the proposition 'A is not C' is true. Again, if the latter be true, then by the Law of Contradiction the proposition 'A is C' is false. Hence, if 'A is B' be true, 'A is C' is false. Similarly, from (3) it can be proved that if 'A is C' be true, then 'A is B' is false. Hence, of 'A is C' and 'A is B,' if one be true, the other is false. Again, if 'A is B' be false, 'A is not-B' is true by the Law of Excluded Middle (*vide* p. 17, and also Ueberweg, pp. 260—3). And if 'A is not-B' be true, then from (2) 'A is C' is true. Similarly, it can be proved that if 'A is C' be false, 'A is B' is true. Hence, of 'A is B' and 'A is C,' if one be false, the other is true. Therefore, of the two propositions 'A is B' and 'A is C,' if one be true, the other is false, and if one be false, the other is true,—that is, they are the two members of the disjunctive proposition 'Either A is B or A is C,' or 'A is either B or C,' in Ueberweg's sense.

(*b*) And from the two hypotheticals may also be inferred the original disjunctive in Mill's sense. The two hypotheticals from the disjunctive, according to Mill, are—

(1) If A is not C, A is B.
(2) If A is not B, A is C.

It has been already shown above that of the two propositions 'A is B' and 'A is C,' the falsity of the one implies the truth of the other—*i.e.*, they are the two members of the disjunctive proposition 'A is either B or C' in Mill's sense.

(*c*) Is it possible to infer immediately a disjunctive proposition from a single hypothetical? This is not possible in Ueber-

weg's sense of a disjunctive. But this is possible of a disjunctive in Mill's sense. From the hypothetical 'If A is B, A is C' follows the disjunctive 'Either A is not B or A is C.' The proof is as follows :—

(1) If A is B, A is C.

By contraposing this we get,

(2) If A is not C, A is not B.

If 'A is C' be false, 'A is not C' is true by the Law of Excluded Middle; and ∴ from (2) 'A is not B' is true. Again, if 'A is not B' be false, 'A is B' is true by the same law; and ∴ from (1) 'A is C' is true. Hence, of the two propositions 'A is C' and 'A is not B,' the falsity of the one implies the truth of the other. They are, therefore, the two members of the disjunctive proposition 'Either A is not B or A is C' in Mill's sense. Thus, a disjunctive in Mill's sense can be inferred from a single hypothetical proposition; but this is not possible in Ueberweg's sense of a disjunctive.

Exercises.

I. Distinguish the following disjunctive propositions from each other, and note the ambiguity, if any, in their meaning :—

1. The individual A is either B or C.
2. An A is either B or C.
3. Some A is either B or C.
4. Every A is either B or C.
5. Either all A is B or all A is C.

II. Infer the hypothetical propositions which follow from each of the above disjunctive propositions in Mill's and also in Ueberweg's sense of a disjunctive.

III. Draw the inferences which follow from the following propositions by change of relation :—

1. Only material bodies gravitate.
2. No plant can grow without light and heat.
3. No animal can live without oxygen.
4. A mineral is either a simple or a compound substance.

5. A material body is either solid or fluid.
6. If a proposition is not real, it is verbal.
7. Hydrogen is either a metal or a non-metal.
8. If a material body is solid, it is not fluid.
9. If mercury is heated, it rises in temperature.

IV. Infer the hypothetical propositions which follow from each of the following disjunctive propositions, and then show that the disjunctive may be re-inferred from them:—

1. Every animal is either vertebrate or invertebrate.
2. The soul is either mortal or immortal.
3. Either no S is P or some S is Q.
4. Either every A is B or some C is not D.
5. Either some A is B or some A is not C.
6. Space is either finite or infinite.
7. Every object of thought is either an idea of sensation or an idea of reflection.
8. An existence is either material or mental.
9. All knowledge is either intuitive or experimental.
10. Every mental phenomenon is either a feeling, a knowing or a willing.
11. A body is either solid, liquid, or gaseous.

V. Distinguish the following disjunctive propositions, and infer the hypothetical propositions which follow from each of them:—

1. The element hydrogen is either a metal or a non-metal.
2. An element is either a metal or a non-metal.
3. Every element is either a metal or a non-metal.
4. Element is either metallic or non-metallic.

VI. Distinguish the disjunctive propositions in each of the following groups, and infer the hypotheticals which follow from each of them:—

1. (a) A substance is either absolute or relative.
   (b) Every substance is either absolute or relative.
   (c) Substance is either absolute or relative.

2. (a) Man is either rational or irrational.
   (b) Every man is either rational or irrational.
   (c) This man is either rational or irrational.

3. (a) This animal is either vertebrate or invertebrate.
(b) Every animal is either vertebrate or invertebrate.
(c) An animal is either vertebrate or invertebrate.

4. (a) Substance is either knowable or unknowable.
(b) A substance is either knowable or unknowable.
(c) All substances are either knowable or unknowable.

5. (a) A body is either solid or fluid.
(b) This body is either solid or fluid.
(c) Every body is either solid or fluid.
(d) All bodies are either solid or fluid.

§ 9. Additional Forms of Immediate Inference.

Given a proposition 'A —[1] B' with 'A' and 'B' as its subject and predicate respectively, the propositions immediately inferred from it will be in one or other of the following forms :—

1. 'A — not-B,' with 'A' and 'not-B' as subject and predicate.

2. 'Not-A — B,' with 'not-A' and 'B' as subject and predicate.

3. 'Not-A — not-B,' with 'not-A' and 'not-B' as subject and predicate.

4. 'B — A,' with 'B' and 'A' as subject and predicate.

5. 'Not-B — A,' with 'not-B' and 'A' as subject and predicate.

6. 'B — not-A,' with 'B' and 'not-A' as subject and predicate.

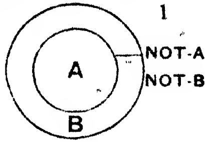
7. 'Not-B — not-A,' with 'not-B' and 'not-A' as subject and predicate.

Of these forms, the 1st is called the obverse, the 4th the converse, the 5th the contrapositive of the given proposition, and these are all that we have recognized and treated of above. But it is evident that the other forms may also be immediately inferred from the given proposition.

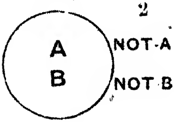
[1] This sign (—) is used in this place to avoid the awkward repetition of the words "is or is not."

On inspection and comparison of the diagrams of A, E, I, O, the following inferences may be easily shown to be legitimate and admissible. In proving these inferences, it is to be remembered that 'A' and 'not-A,' and 'B' and 'not-B,' cover the whole sphere of thought and existence (*vide* pp. 51—52)[1]:—

1

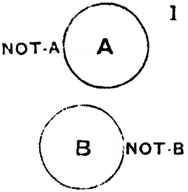


2

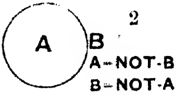


I.—From A "All A is B" follow:—
(1) No A is not-B (E, obverse).
(2) Some not-A is not B (O).
(3) Some not-A is not-B (I).
(4) Some B is A (I, converse).
(5) No not-B is A (E, contrapositive).
(6) Some B is not not-A (O).
(7) All not-B is not-A (A).

1



2



II.—From E "No A is B" follow:—
(1) All A is not-B (A, obverse).
(2) Some not-A is B (I).
(3) Some not-A is not not-B (O).
(4) No B is A (E, converse).
(5) Some not-B is A (I, contrapositive).
(6) All B is not-A (A).
(7) Some not-B is not not-A (O).

III.—From I "Some A is B" follow:—
(1) Some A is not not-B (O, obverse).
(4) Some B is A (I, converse).
(6) Some B is not not-A (O).

[1] It is assumed that every term, whether subject or predicate of a proposition, has a term contradictory to it. See Appendix G.

IV.—From O "Some A is not B" follow:—

(1) Some A is not-B (I, obverse).
(5) Some not-B is A (I, contrapositive).
(7) Some not-B is not not-A (O).

The other forms in the case of I and O are wanting.

Of the seven forms given above, three—(1), (4), and (5)—have, as we have already stated, special names: obverse, converse, and contrapositive respectively; the others—(2), (3), (6), and (7)—have no special names. That these inferences are valid may be easily proved also by the older method. For example, of the inferences drawn from A, (7) is the obverse of its contrapositive, (6) is the obverse of its converse, (3) is the converse of the obverse of its contrapositive, and (2) is the obverse of (3). Of the inferences drawn from E, (2) is the contrapositive of its converse, (3) is the obverse of (2), (6) is the obverse of its converse, and (7) is the obverse of its contrapositive. Thus the four additional forms may be inferred by the older method as well as by the method adopted in this work,—by the former as an inference from an inference, and by the latter as an immediate inference from the given proposition.

§ 10. Miscellaneous Exercises.

I. Give the obverse of the converse of the following propositions:—

(1) The useful is not the beautiful.
(2) Beauty is unity in variety.
(3) Wise men are few.
(4) A touches B.
(5) (*a*) I know, (*b*) I am, (*c*) He is.
(6) A is equal to B.
(7) A lies above B.
(8) The number of substances containing more than four elements is very small.
(9) Where no object is distinguished, we are not conscious of any.

(10) A is greater than B.
(11) A strikes B.
(12) A includes B.

II. Test the following inferences:—

1. Cold is agreeable;
∴ Heat is disagreeable.
2. Some elements are metals;
∴ Some non-metal is element.
3. If a body is heated, it will expand;
∴ If a body expands, it is heated.
4. Some plants can move .... is true;
∴ Some plants can not move .... is also true.
5. If the rays of light fall upon the eye, they will produce the sensation of vision;
∴ If the sensation of vision is not produced, the rays of light have not fallen upon the eye.
6. All A is B.
∴ Some not-A is not-B.

III. Give the converse of the contradictory of each of the following propositions:—

1. Every man is not learned.
2. Only animals are sentient beings.
3. Nothing is annihilated.
4. If A is B, C is not D.

IV. Give the contrapositive of the contrary of each of the following propositions:—

1. Every phenomenon has a cause.
2. No man is perfect.
3. If A is B, C is D.
4. If A is B, C is not D.

V. Give the converse of the contrapositive of the contrary or subcontrary of the contradictory of each of the following propositions:—

1. All sensations are feelings.
2. No man is immortal.
3. Some men are wise.
4. Some elements are not metals.

VI. Given the proposition 'Some men are not selfish' as true: state the propositions that can be inferred from it, (1) as true, (2) as false, and (3) as doubtful or unknown.

VII. Given the proposition 'The virtuous are happy' as true: state the propositions that can be inferred from it, (1) as true, (2) as false, and (3) as doubtful or unknown.

VIII. Given the proposition 'Some men are unjust' as true: state the propositions that can be inferred from it, (1) as true, (2) as false, and (3) as doubtful or unknown.

IX. Given the proposition 'No man is infallible' as true: state the propositions that can be inferred from it, (1) as true, (2) as false, and (3) as doubtful or unknown.

X. Infer as many verbal or analytical propositions as you can from each of the following terms:—(1) animal, (2) matter, (3) triangle, (4) circle, (5) square, (6) man, (7) plant, (8) metal, (9) force, (10) book, (11) table, (12) horse, (13) mammal, (14) mind, (15) perception, (16) sensation, (17) house, (18) philosopher, (19) poet, (20) king, (21) nation, (22) society, (23) paper, (24) chair, (25) examination.

XI. Draw as many inferences as you can from the truth and also from the falsity of each of the following propositions:—

(1) All S is P.

(2) No S is P.

(3) Some S is P.

(4) Some S is not P.

XII. Infer as many propositions as you can from each of the following propositions being given as true:—

(1) Every phenomenon has a cause.

(2) The invariable antecedent of a phenomenon is the cause of the phenomenon.

(3) The absolute commencement of a phenomenon is not conceivable.

(4) The infinite non-commencement of a phenomenon is not conceivable.

(5) At least one substance has no cause.

CHAPTER III.

Of Syllogisms.

§ 1. A Syllogism is the inference of a proposition from two given propositions, the inferred proposition being not more general than either of the two given propositions. As an argument fully expressed in language, it consists of three propositions, one of which, the conclusion, follows necessarily from the other two, called the Premisses, and thus differs from Immediate Inference, which, as the simplest and most elementary form of argument, consists of two propositions, the conclusion and the proposition from which the conclusion necessarily follows. From the proposition 'All men are mortal' follows 'Some mortal beings are men' by immediate inference,—*i. e.*, the latter is a conclusion derived from the former without the aid of any other proposition. In a Syllogism such aid is necessary, that is, a conclusion is drawn not from one proposition but from at least two propositions. For example, from the two propositions 'All men are mortal' and 'Philosophers are men,' I infer the proposition 'Philosophers are mortal.' Here (1) the conclusion follows from the two propositions taken jointly, and not from either of them singly. The two propositions must be brought together before I can legitimately infer the third which is involved in them, and yet is distinct from either. The conclusion 'Philosophers are mortal' is not the same as either of the two propositions 'All men are mortal' and 'Philosophers are men'; nor does it follow from one of them. By this character a syllogism is distinguished from an immediate inference. Again, (2) the two propositions being true, the conclusion must

be true. The one conjointly with the other makes the conclusion necessarily admissible, legitimate, or valid. By this character a syllogism, that is, a correct or valid syllogism, is distinguished from an apparent one or a mere combination of three propositions in which the conclusion does not follow from the premisses. And (3) the conclusion can not be more general than either of the two propositions from which it is inferred. The proposition 'Philosophers are mortal' is less general than the proposition 'All men are mortal,' the latter being applicable to a much larger number of individual things than the former. By this character, a syllogism is distinguished from an induction, in which we pass from the less general to the more general, from the particular to the universal[1].

A syllogism is either pure or mixed. It is pure when both its premisses have the same relation, that is, when they are both categorical or both hypothetical; and mixed when they have different relations, that is, when one of them is hypothetical and the other categorical, or one disjunctive and the other categorical. These distinctions will be referred to more fully in a subsequent chapter[2].

§ 2. Of Categorical Syllogisms.

A Categorical Syllogism is a syllogism consisting of two categorical premisses and a categorical conclusion necessarily following from them. It is a reasoning in which a term is affirmed or denied of another by means of a third. Given two terms: if I affirm or deny one of the other, I get a categorical proposition 'A is B' or 'A is not B.' In this act there is no reasoning, mediate or immediate; there is merely an act of judgment, the direct comparison of one term with the other. If every term could be thus directly affirmed or denied of every other, there would be no such mental act as reasoning; there would be no need of it. But constituted and circumstanced as we are, we can not directly affirm or deny every term of every other. We have often to establish a relation between two terms

[1] See above, Part III, Chap. I.

[2] See below, Part III, Chap. V.

from the relation which each of them bears to a third. Given, say, two terms 'A' and 'C': it is required to find out whether A is to be affirmed or to be denied of C. Failing to do this by immediate comparison, I affirm A of every B, and B of every C, and therefrom affirm A of every C. The reasoning is thus expressed in the form of a categorical syllogism :—

Every B is A,
Every C is B;
∴ Every C is A.

In this reasoning I really compare the whole of B with A, and the whole of C with B, and thus establish a relation between the whole of C and A. I find, for example, that all the things called 'B' are included in the things called 'A,' and that all the things called 'C' are included in the things called 'B,' and conclude therefrom that all the things called 'C' are included in the things called 'A'; or I find that A-things co-exist with B-things, and that the latter co-exist with C-things, and conclude therefrom that the first co-exist with the last.

The two terms 'A' and 'C,' of which one is affirmed or denied of the other in the conclusion, are called the Extremes, while the third term B, with which each of them is compared, is called the Middle Term. The extremes occur in the premisses as well as in the conclusion, while the middle term occurs in the premisses only. The extreme, which is the subject in the conclusion, is usually called the Minor Term, and that which is the predicate, the Major Term; the premiss which contains the minor term, the Minor Premiss, and that which contains the major term, the Major Premiss.

§ 3. Whether a particular combination of three propositions constitutes a valid syllogism or not, may be easily ascertained with the aid of the diagrams used in explaining immediate inference in the preceding chapter. Draw the diagrams representing the major premiss and combine with each of them every diagram representing the minor premiss, and if the conclusion follows from each combination, then the three propositions

constitute a valid syllogism; if not, not. If the major or the minor premiss is represented by a single diagram, then combine this one with each diagram representing the other premiss, and if the conclusion follows from each combination, then the three propositions constitute a valid syllogism; if not, not. In this same way we may ascertain whether two premisses lead to any conclusion; and if so, to what conclusion. In this method of testing syllogisms, we use the following two axioms :—

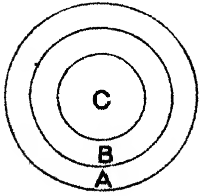
(1) Two circles coinciding with a third by any the same part coincide with each other by that part.

(2) Two circles of which one coincides and the other does not with a third by any the same part do not coincide with each other by that part.

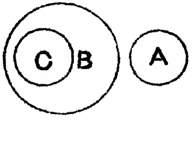
When the first axiom is applicable, the conclusion is affirmative; when the second is applicable, the conclusion is negative; and when neither is applicable, there is no conclusion.

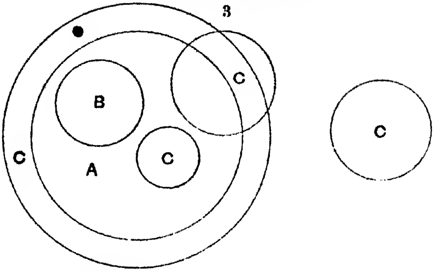
The truth of these axioms is evident to every person who understands the meaning of the words in which they are expressed. "Any the same part" may be "the whole" or "the smallest part possible." And the part with which one coincides may be either a part or the whole of the part with which the other coincides or does not coincide. The meaning of the words may be further illustrated by the following diagrams :—

1



2





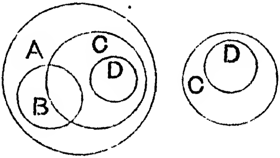
In the first diagram, two circles A and C coincide with B by any the same part,—namely, the whole of C or a part of A; therefore they coincide with each other by that part, that is, "all C is A" or "some A is C." This diagram is, in fact, a representation of the syllogism "all B is A, all C is B; therefore all C is A," and also of the syllogism "all C is B, all B is A; therefore some A is C."

In the second diagram, of the two circles C and A, C coincides with a third B by a part (the whole of C), and the other A does not coincide with B by the same part (the whole of C); therefore they do not coincide with each other by that part, that is, "no A is C," or "no C is A." This diagram is, in fact, a representation of the syllogism "all C is B, no A is B; ∴ no A is C," and also of the syllogism "no A is B, all C is B; ∴ no C is A."

In the third diagram no conclusion follows, because neither axiom is applicable to it, the circle C lying either outside or inside of the circle A or including it.

§ 4. By these two axioms we can distinguish a categorical syllogism, that is, a valid categorical syllogism from an apparent one, or a mere combination of three propositions in which the conclusion does not follow from the premisses. But to help the student still further in this most important process of testing syllogisms, we shall give below certain rules to which every categorical syllogism must conform. These Syllogistic Rules follow from the definition of a categorical syllogism:—

1. *Every categorical syllogism must contain three and only three terms*, neither more nor less,—namely, the two extremes between which we find a relation, and the third or middle term with which we compare each extreme in order to compare them with each other. If there be less than three, there is no means of finding the relation between the two extremes. If there be more, either there is a train of reasoning consisting of a series of syllogisms, or there is no reasoning at all. "A is B, B is C, C is D; therefore A is D." Here there are four terms, and there is a series of two syllogisms. The first two propositions give the conclusion 'A is C,' and this proposition and the next, namely, 'C is D,' allow the conclusion 'A is D.' But the following propositions containing four terms do not constitute any reasoning: "A is B, C is D, B is A, and D is C." Here there are four propositions, from which we can not infer any relation between A and C or D, or between B and C or D. This will be evident from the following figures representing the last two propositions:—



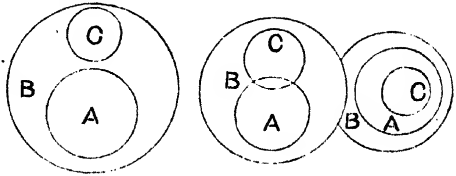
A and B may or may not lie outside C or D, that is, their relation is unknown, and can not be determined from those two propositions. It follows from this rule that no term should be ambiguous; for an ambiguous term having two distinct meanings is really equivalent to two terms, and the three terms are, in that case, really equivalent to four.

2. *Every categorical syllogism, when fully expressed, contains three and only three propositions*,—namely, the two premisses in which the middle or third term is compared with each of the two extremes, and the conclusion which expresses a relation

between the extremes, and which follows necessarily from the two premisses.

3. *The middle term must be distributed at least once.* This rule and those which are given below, follow from that part of the definition of the syllogism which requires that the conclusion must necessarily follow from the premisses. The present rule means that the middle term with which the two extremes are compared, must be taken once at least in its universal or entire extent. In other words, the whole of the circle standing for the middle term must at least once be compared with either of the two circles representing the two extremes; for otherwise one extreme might be compared with one part of the middle term, and the other with another part of it, in which case no comparison could be possible between the two extremes. This will be evident from the following diagrams:—

All A is B.
All C is B.
No conclusion.



All A and all C are each compared with a part of B, and from these two comparisons we can draw no conclusion as to the relation between C and A, that is, we can not infer that A lies outside of C, or that it lies inside of C, or that A and C intersect. This is evident from the three cases represented above. The violation of this rule leads to a fallacy, technically called the Fallacy of Undistributed Middle.

4. *No term must be distributed in the conclusion which was not distributed in one of the premisses.* The non-distribution of a term in one of the premisses means that its extent has not been definitely expressed, that it has not been exactly stated whether the whole or part of its extent is meant, and that all that has been said about it is, that at least one individual or case has

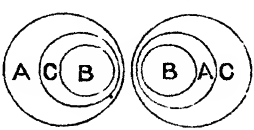
been taken into consideration, while the whole is not excluded[1]. From this vagueness and indefiniteness about the extent of the term in one of the premisses, we can not, in the conclusion, take the term in its entire extent, *i.e.*, distributively. In some cases this may be allowed; but in other cases this can not be; so generally we can not distribute a term in the conclusion unless it is distributed in one of the premisses. For it must not be forgotten that what we are allowed to infer in mediate as well as in immediate inference, is not that which follows in one or two cases, but that which follows in all cases, and that if a proposition does not follow equally in all cases, it can not be regarded in Logic as a legitimate inference. This will be evident from the following diagrams:—

All B is A,

All B is C,

∴ All C is A.

1 2



From the first diagram the conclusion follows. But from the second, which also represents the premisses, it does not follow. Hence the conclusion in the general form is not true. C not being distributed in the second premiss, can not be distributed in the conclusion. The correct conclusion is 'Some C is A.'

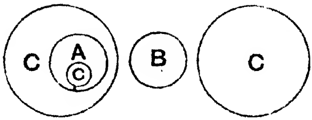
The violation of this rule leads to a fallacy, technically called the Fallacy of Illicit Process, either of the subject or of the predicate in the conclusion, that is, of the minor or of the major term.

5. *If both the premisses be negative, nothing can be inferred.* For what is expressed in the premisses is that there is no connection between the middle term and each of the two extremes; and from this nothing can be inferred between the two extremes themselves—they may or may not be connected with each other.

[1] See above, Part II, Chap. III.

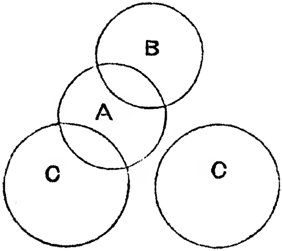
This can easily be proved by the comparison of the diagrams. A negative premiss is represented by the 3rd, 4th, and 5th diagrams.

Take the 4th and 4th. Here no conclusion follows. A and C may include each other or lie outside each other.



4th and 4th.

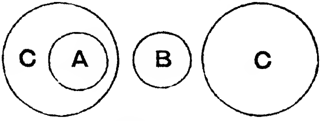
Take the 3rd and 4th. Here A and C either lie outside each other or intersect with each other, and we may infer 'Some A is not C,' but as this conclusion does not follow in the other cases, we can not infer it generally.



3rd and 4th.

Or we may prove the rule thus. The negative premisses must be either EE, EO, or OO in any order; and it will be seen, on the comparison of the diagrams, that no conclusion follows generally from any of these combinations of premisses, *i.e.*, from each particular case of each combination. A conclusion may

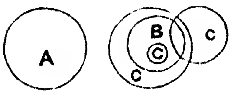
follow in *one* case of a combination, but if it does not follow in the *other* cases, it can not be regarded as a legitimate conclusion of that combination. The following diagram represents a case,



4th and 4th.

namely, 4th and 4th, of each of the three combinations; and from this no conclusion follows, as we have already seen.

6. *If one premiss be negative, the conclusion must be negative.* That is, in those cases in which the conclusion does follow, it must be negative; for there may be cases in which no conclusion follows. The negative premiss merely expresses that there is no connection between the middle term and one of the extremes, and the other premiss, which must be affirmative, expresses that there is some connection between the middle term and the other extreme. From this all that we can infer is, that there is no connection between the two extremes. The negative premiss may be represented by two circles A and B lying outside each other, and the affirmative premiss by the circle B and another C, either including each other, or intersecting, or coinciding with each other. In all these different cases a part of C must be within B, which lies outside A.



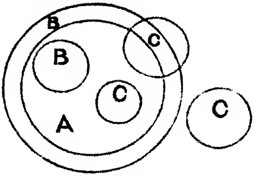
Hence we may infer that a part of C lies outside A, or "Some C is not A," a negative conclusion.

To prove the rule more satisfactorily we may have recourse to the following method. The possible premisses are AE, AO, IE, IO in any order. It will be seen from the comparison of the diagrams that in those cases in which a conclusion follows, the conclusion is negative.

Take, for example, the combination AE. It has the following different cases:—

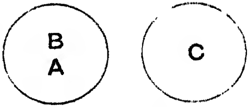
The 1st diagram and 4th, 2nd and 4th, 4th and 1st, and 4th and 2nd.

From the 1st and 4th follows a negative conclusion, namely, 'Some A is not C.'



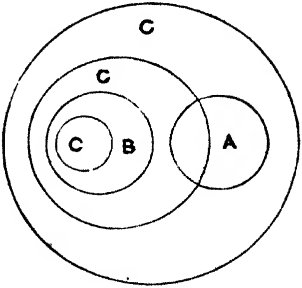
1st and 4th.

From the 2nd and 4th follows a negative conclusion, namely, 'No C is A.'



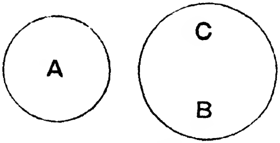
2nd and 4th.

From the 4th and 1st follows a negative conclusion, namely, 'Some C is not A.'



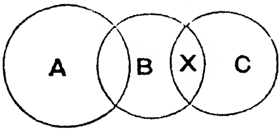
4th and 1st.

From the 4th and 2nd follows a negative conclusion, namely, 'No C is A.'



4th and 2nd.

Conversely, it can be shown that *to prove a negative conclusion one of the premisses must be negative.* A negative conclusion means that there is no connection between the two extremes, and this can only be proved by a premiss which expresses that there is no connection between the middle term and one of the extremes, and a premiss which expresses that there is a connection between the middle term and the other extreme, *i.e.*, by a negative and an affirmative premiss. A negative conclusion, for example, 'Some C is not A' means that at least a part of C lies outside the whole of A. In order to prove this, the following premisses are necessary,—1*st*, that a part of C coincides with a part of B, and 2*ndly*, that the part of B which coincides with a part of C lies outside the whole of A, the first being an affirmative and the second a negative premiss.



Here the *crossed* part of C coincides with the crossed part of B that lies outside the whole of A, therefore the crossed part of C lies outside the whole of A.

7. *If both the premisses are affirmative, the conclusion must be affirmative.* For, if the conclusion be negative, one of the premisses must be negative by the converse of Rule 6 ; but both

the premisses are, by supposition, affirmative; therefore the conclusion must be affirmative. Conversely, it can be shown that *to prove an affirmative conclusion, both the premisses must be affirmative.* For, if one of the premisses be negative, the conclusion will, by Rule 6, be negative; therefore both the premisses must be affirmative.

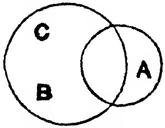
8. *If both the premisses be particular, nothing can be inferred.* The two particular premisses are either II, IO, or OO in any order. In the first combination the middle term is not distributed in either of the premisses. In the second, it may be distributed by being the predicate in O, but as the conclusion must be negative, a term will be distributed, also, in the conclusion, which was not distributed in the premisses; hence there will be an illicit process either of the subject or of the predicate in the conclusion. No conclusion follows from the last combination, both the premisses being negative. Hence it is true universally that nothing can be inferred if both the premisses be particular.

9. *If one of the premisses be particular, the conclusion must be particular.* If one premiss be particular, the other must be universal, for from two particular premisses nothing can be inferred.

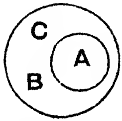
Hence, the two premisses are either IA, or IE, or OA, or OE in any order. The conclusion of IA or AI must be particular, because in the premisses only one term (the subject in A) is distributed, and that, therefore, must be the middle term; and if the conclusion were universal, a term would be distributed in it which was not distributed in the premisses; hence there would be an illicit process. The conclusion of IE or EI must be particular, for if it were universal, there would be, as in the preceding case, an illicit process. In the premisses two terms only are distributed; of these one must be the middle term, and the other one only, therefore, can be distributed in the conclusion. But the conclusion must be negative, as one of the premisses is negative, and if it were, also, universal, both its subject and predicate would be distributed; and hence there would be a

term distributed in the conclusion, which was not distributed in the premisses. Similarly, the conclusion from OA or AO must be particular; only two terms are distributed in the premisses; of these one must be the middle term, and the other the predicate of the conclusion, which will be negative, and have, therefore, the predicate distributed. Hence the subject of the conclusion must be undistributed, that is, the conclusion must be particular; otherwise there would be an illicit process. No conclusion follows from OE, as both the premisses are negative.

This rule can also be proved from the diagrams. Take the combination IA. From the 3rd and 2nd diagrams follows a



3rd and 2nd.



1st and 2nd.

particular conclusion, 'Some C is A,' and from the 1st and 2nd follows a particular conclusion, 'Some C is A.' In some cases, as in the 2nd and 2nd, a universal may follow; but as this does not follow in the other cases, it is inadmissible.

From this rule, it is evident that *if the conclusion is universal, both the premisses must be universal.* For, if one of the premisses be particular, the conclusion will be particular. Therefore both the premisses must be universal.

The last three rules, *viz.*, the 7th, 8th, and 9th, are merely consequences of the other rules. A violation of any of those three rules is a result of the violation of some of the other rules. If the other rules are carefully observed, the last three must be observed along with them, and can not be violated.

§ 5. Division of Categorical Syllogisms into Figures.

Every valid categorical syllogism must conform to the nine rules, or conditions laid down and proved above. By the help of those rules, we can easily distinguish a valid from an invalid

categorical syllogism. Given any combination of two premisses and a conclusion, we can, by the aid of the rules, determine whether the conclusion follows from the premisses or not. When only two premisses are given, we can determine whether they lead to any conclusion, and if so, to what conclusion.

In every categorical syllogism there must be two premisses and a conclusion determined by the premisses. Given the premisses, the nature of the logitimate conclusion is given along with them. In the premisses, the middle term may have different positions in different syllogisms, and the primary division of categorical syllogisms is founded on the difference in position of the middle term in relation to the extremes in the premisses. The division is into three classes, technically called *Figures*, and is as follows:—

(1) The middle term is the subject in one premiss, and predicate in the other.

(2) The middle term is the predicate in both the premisses.

(3) The middle term is the subject in both the premisses.

Taking B to be the middle term and A and C the extremes, the three classes may be thus symbolically expressed:—

1st Class.	2nd Class.	3rd Class.
B A	A B	B A
C B	C B	B C
∴ C A or A C.	∴ C A or A C.	∴ C A or A C.

The conclusion expresses a relation between C and A, and is represented by a proposition whose subject and predicate are either A and C or C and A respectively.

If we always take C as the subject and A as the predicate in the conclusion, and call them the minor and the major term, and the two premisses in which they occur the minor and the major premiss respectively[1], we get *four classes or Figures* as follows:—

[1] It should be observed that the distinction between the major and the minor term is purely conventional. There is no reason why the subject of the conclusion should be called the minor and the predicate the major term. It is due to usage that the two names 'minor term'

1st.	2nd.	3rd.	4th.
B A	A B	B A	A B
C B	C B	B C	B C
∴ C A	∴ C A	∴ C A	∴ C A

(1) In the 1st figure the middle term is the subject in the major premiss, and predicate in the minor premiss.

(2) In the 2nd, the middle term is the predicate in both the premisses.

(3) In the 3rd, the middle term is the subject in both the premisses.

(4) In the 4th, the middle term is the predicate in the major premiss and subject in the minor.

The conclusion is always a proposition, having C and A respectively for its subject and predicate.

The first classification or division is founded on the difference in position of the middle term in the premisses. The second is founded on this difference and on the distinction between the predicate and the subject in the conclusion, or between the major and the minor term, and the consequent distinction between the major and the minor premiss.

On the first method of classification of syllogisms there are three Figures, and on the second method there are four. On the first method the conclusion is of the form C A or of the form A C; and, on the second method, it is always of the form C A. As best adapted for teaching and as sanctioned by high authorities, we shall adopt here the four-fold classification, and take the conclusion to be always of the form C A[1].

and 'major term' are applied to the subject and the predicate, respectively, in the conclusion. The definition of the minor term is that it is the subject, and the definition of the major term is that it is the predicate, in the conclusion; in other words, the term that is the subject in the conclusion is defined as the minor term, and the term that is the predicate as the major term of a syllogism.

[1] Some logicians obtain the four figures by a double division. Ueberweg, for example, first divides all categorical syllogisms into

§ 6. Subdivision of Categorical Syllogisms in each Figure into Moods.

A syllogism may differ from another not only in the position of the middle term in the premisses, but also in the quantity and quality of the two premisses themselves. Each of the two premisses of a syllogism in each figure may consist of any one of the four propositional forms A, E, I, and O. The major premiss may be any one of these four forms, and the minor, again, may be any one of them. Thus there may be sixteen possible combinations of premisses in each figure, the first letter in each combination representing the major premiss, and the second letter the minor premiss, of a possible syllogism :—

A A	E A	I A	O A
A E	E E	I E	O E
A I	E I	I I	O I
A O	E O	I O	O O

Theoretically there can not be any other combination of premisses. All possible ones are enumerated in the list above. Each of these combinations does not however lead to a valid conclusion, and does not, therefore, constitute a valid syllogism. By the rules given above, and by the method of the comparison of the diagrams, we shall now test these combinations, and find out which of them yield valid forms of syllogism, technically called *Moods*, and which do not, in each figure.

Of the sixteen combinations we may at once reject EE, EO, OE, and OO as invalid in all figures, because no conclusion

three chief classes, called Figures in the more comprehensive sense (the three-fold classification given above), and then subdivides the first of these three classes into two according as the middle term is the subject in the major premiss and predicate in the minor, or the predicate in the major premiss and subject in the minor, the former subdivision corresponding to the first, and the latter to the fourth of the four-fold classification given above. The second and third primary classes do not give rise to any subdivisions. The four classes thus obtained by a double division are called by him Figures in the narrower sense.

follows from two negative premisses (Rule 5). We may also reject II, IO, OI as invalid, because nothing can be inferred from two particular premisses (Rule 8).

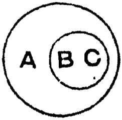
We shall now see what conclusions the remaining nine combinations AA, AE, AI, AO, EA, EI, IA, IE, and OA lead to, and which of them yield valid forms of syllogisms or moods, and which do not, in each figure.

§ 7. Valid Moods in the First Figure.

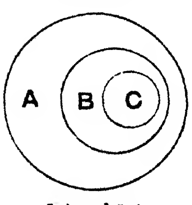
1. Take AA :—The conclusion is A. For by Rule 7, it must

A. All B is A,
A. All C is B;
A. ∴ All C is A[1].

be affirmative, *i.e.*, A or I; and as no rule is violated by inferring A in this case, it is A. That AA gives A as the conclusion in the 1st figure can be proved from the diagrams, thus:—The major premiss A is represented by the 1st and the 2nd diagram. The minor premiss A is represented by the same two diagrams.



1st and 2nd.



1st and 1st.

Combine each of the one with each of the other, and draw the conclusion which follows from each combination, remembering that C must be the subject, and A the predicate, in the conclusion. There are four cases, namely, the 1st and 2nd, 1st and 1st, 2nd and 1st, and 2nd and 2nd. From 1st and 2nd follows A 'All C is A.' From 1st and 1st also follows A. Similarly, from the other two cases of AA in the 1st figure follows A. AAA is, therefore, a valid mood in the first figure. From A follows I by subalternation, or I may be inferred directly from the diagrams.

2. Take next AE :—No conclusion follows. For by Rule 6,

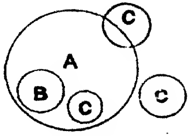
[1] It should be remembered that in this and in the examples that follow, B is taken as the middle term, A as the major term, C as the minor term, and CA as the typical form of the conclusion.

it must be negative, *i.e.*, E 'No C is A,' or O 'Some C is not A'; but as in E and O, the major term A is distributed, while it is undistributed in the major premiss, that is, as Rule 4 is violated by inferring E or O in this case, no conclusion follows.

A. All B is A.
E. No C is B,
No conclusion.

This can be proved from the diagrams. The major premiss A is represented by the 1st and 2nd diagrams, and the minor premiss E by the 4th diagram.

From 1st and 4th no conclusion follows, because C may be outside or inside A.



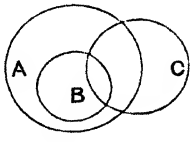
1st and 4th.

3. Take next AI :—The conclusion is I. For by Rules 7 and 9, it can not be anything else than I; and as no rule is violated by inferring I in this case, it is I.

This can be proved from the diagrams. The major premiss A is represented by the 1st and 2nd diagrams; and the minor premiss I by the 1st, 2nd, 3rd, and 5th diagrams. Combine each of the one with each of the other, and draw the conclusion of the form CA, which follows from each combination.

A. All B is A,
I. Some C is B;
I. ∴ Some C is A.

From the 1st and 3rd follows 'Some C is A' (I). Similarly, from the 1st and 5th, 1st and 1st, 1st and 2nd, 2nd and 3rd, 2nd and 5th, 2nd and 2nd, and 2nd and 1st follows also the same conclusion.



1st and 3rd.

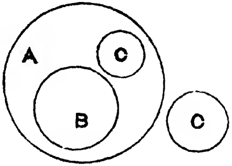
The student should draw these diagrams, and satisfy himself that the conclusion really follows from them.

4. AO :—No conclusion follows. For by Rules 6 and 9, it can not be anything else than O; but as in O 'Some C is not A,' A is distributed, while it is undistributed in the

A. All B is A,
O. Some C is not B.
No conclusion.

major premiss, that is, as Rule 4 is violated by inferring O in this case, no conclusion follows.

From the 1st and 4th diagrams, representing the major premiss A and the minor premiss O, respectively, nothing follows, because C may be outside or inside A.

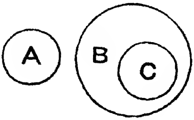


1st and 4th.

5. EA:—The conclusion is E. For by Rule 6, it must be negative, *i.e.*, E or O; and as no rule is violated by inferring E in this case, it is E.

E. No B is A,
A. All C is B;
E. ∴ No C is A.

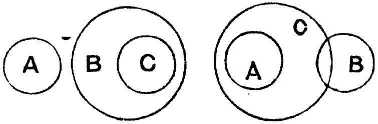
From the 4th and 1st follows E 'No C is A.' From the 4th and 2nd also follows 'No C is A.' From E follows O 'Some C is not A' by subalternation, or O may be inferred directly from the diagrams.



4th and 1st.

6. EI:—The conclusion is O. For by Rules 6 and 9, it can not be anything else than O; and as no rule is violated by inferring O in this case, it is O. From the 4th and 1st as also from the 4th and 2nd, 4th and 3rd, and 4th and 5th follows Some C is not A (O).

E. No B is A,
I. Some C is B;
O. ∴ Some C is not A.



4th and 1st. 4th and 3rd.

7. IA:—No conclusion can be drawn from this by Rule 3, because the middle term B is not distributed, being the predicate in A and the subject in I.

8. IE:—No conclusion follows. For by Rules 6 and 9, it can not be anything else than O 'Some C is not A'; but as in O, the term A is distributed in the conclusion, while it is undistributed in the major premiss, that is, as Rule 4 is violated by inferring O in this case, no conclusion follows.

9. OA:—Here the middle term is not distributed, and hence no conclusion can be drawn according to Rule 3.

In the first figure or class the combinations AA, AI, EA, and EI lead, then, to valid conclusions, and yield the following valid forms of syllogisms or *moods:* AAA, AII, EAE, EIO, technically called *Barbara, Darii, Celarent,* and *Ferio.* The conclusions of the moods AAI and EAO, which are also valid, may be inferred from the conclusions of AAA and EAE by subalternation. Hence they have been called subaltern moods, and are quite useless.

By comparing these valid moods with one another we can generalize the following two special rules of the first figure:—

(1) The major premiss must be universal. This is true of every one of the valid moods.

(2) The minor premiss must be affirmative. This is also true of every one of them.

These two special rules of the first figure may be proved thus by the general syllogistic rules. If the minor premiss be negative, the major premiss must be affirmative by Rule 5, and the conclusion negative by Rule 6, *i.e.*, A will be distributed in the conclusion, being the predicate in a negative proposition, when it

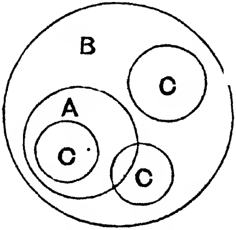
has not been distributed in the major premiss, being the predicate in an affirmative proposition. Hence the minor can not be negative; it must, therefore, be an affirmative proposition. Secondly, if the major be particular, the middle term B will not be distributed in the premisses, being the subject in a particular proposition, and predicate in an affirmative proposition. The major premiss must, therefore, be universal.

§ 8. Valid Moods in the Second Figure.

1. AA:—Nothing follows, because the middle term B is not distributed, being the predicate in two affirmative propositions.

A. All A is B,
A. All C is B,
No conclusion.

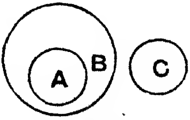
From the 1st and 1st diagrams representing the major and the minor premiss A respectively, nothing follows, because C might be inside or outside A.



1st and 1st.

2. AE:—The conclusion is E. For by Rule 6, it must be negative, *i.e.*, E or O; and as no rule is violated by inferring E in this case, it is E. This can be proved from the diagrams. The major premiss A is represented by the 1st and 2nd diagrams; and the minor premiss E by the 4th. Combine these in the usual way. From the 1st and 4th diagrams follows E 'No C is A.' From the 2nd and 4th also E follows. AEE is, therefore, a valid form

A. All A is B,
E. No C is B;
E. ∴ No C is A.



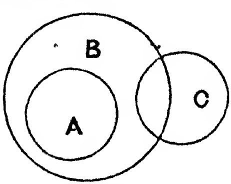
1st and 4th.

of syllogism or mood in the 2nd figure. From E follows O by subalternation, or O may be inferred directly from the diagrams.

3. AI:—Nothing follows, because the middle term is not distributed.

4. AO:—The conclusion is O. For by Rules 6 and 9, it can not be anything else than O; and as no rule is violated by inferring O in this case, it is O. The major premiss A is represented by the 1st and 2nd diagrams; and the minor premiss O by the 3rd, 4th, and 5th. Combine each of the one with each of the other.

A. All A is B,
O. Some C is not B;
O. ∴ Some C is not A.

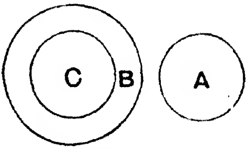


1st and 3rd.

From the 1st and 3rd diagrams follows O 'Some C is not A'; similarly, from the 1st and 4th, 1st and 5th, 2nd and 3rd, 2nd and 4th, 2nd and 5th also follows O. AOO is, therefore, a valid form of syllogism or mood in the 2nd figure.

5. EA:—The conclusion is E. For by Rule 6, it must be negative, *i.e.*, E or O; and as no rule is violated by inferring E in this case, it is E.

E. No A is B,
A. All C is B;
E. ∴ No C is A.



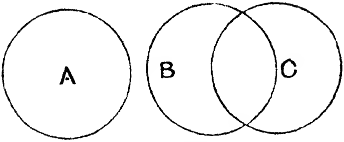
4th and 1st.

From the 4th and 1st follows E 'No C is A,' in the 2nd figure. Similarly from the 4th and 2nd follows E. EAE is, therefore, a valid form of syllogism or mood in the second figure. From E follows O by subalternation, or O may be inferred directly from the diagrams.

6. EI:—The conclusion is O. For by Rules 6 and 9, it can not be anything else than O; and as no rule is violated by inferring O in this case, it is O.

E. No A is B,
I. Some C is B;
O. ∴ Some C is not A.

From the 4th and 3rd follows O 'Some C is not A.' The part lying within B must be outside A.



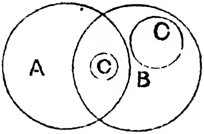
4th and 3rd.

Similarly, from the 4th and 2nd, 4th and 1st, 4th and 5th follows O 'Some C is not A.' EIO is, therefore, a valid form of syllogism or mood in the second figure.

I. Some A is B,
A. All C is B,
No conclusion.

7. IA :—Nothing can be inferred because the middle term is not distributed in the premisses.

From the 3rd and 1st nothing follows, for C may lie outside or inside A.



3rd and 1st.

8. IE :—No conclusion follows. For by Rules 6 and 9, it can not be anything else than O; but as Rule 4 is violated by inferring O in this case, no conclusion follows.

9. OA :—Nothing follows for the same reason as in the preceding case.

The valid forms of syllogism or moods in the second figure are, therefore, AEE, AOO, EAE, and EIO, technically called *Camestres*, *Baroko*, *Cesare*, and *Festino*. AEO and EAO are also valid, being merely the weakened forms of AEE and EAE; as their conclusions follow by subalternation from those of the latter, they are called subaltern moods.

From these valid moods we can generalize the following special rules of the second figure:—

(1) The major premiss must be universal.

(2) One of the two premisses must be negative.

(3) The conclusion must be negative.

Each of these rules holds good in each of the valid moods. They may be thus proved by the general syllogistic rules. If one of the premisses be not negative, the middle term will not be distributed. If one premiss be negative, the conclusion must be negative by Rule 6. The conclusion being negative, the major term, which is the predicate in it, is distributed, and must, therefore, be also distributed in the premisses; and this will not be the case, unless the major premiss be universal, because the major term is the subject in this premiss.

§ 9. Valid Moods in the Third Figure.

1. Take AA:—The conclusion is I. For by Rule 7, it must be affirmative, *i.e.*, A or I; but as Rule 4 is violated by inferring A, it can not be A; and as no rule is violated by inferring I in this case, it is I.

A. All B is A,
A. All B is C;
I. ∴ Some C is A.

AAI is, therefore, a valid mood in the 3rd figure.

2. AE:—No conclusion follows. For by Rule 6, it must be negative; and as Rule 4 is violated by inferring a negative conclusion in this case, no conclusion follows.

A. All B is A,
E. No B is C,
No conclusion.

3. AI:—The conclusion is I. For by Rules 7 and 9, it can not be anything else than I; and as no rule is violated by inferring I in this case, it is I.

A. All B is A,
I. Some B is C;
I. ∴ Some C is A.

4. AO:—No conclusion follows for the same reason as in the case of AE.

A. All B is A,
O. Some B is not C,
No conclusion.

5. EA:—The conclusion is O. For by Rule 6, it must be negative, *i.e.*, E or O; but as Rule 4 is violated by inferring E, it can not be E; and as no rule is violated by inferring O in this case, it is O.

6. EI :—The conclusion is O. For by Rules 6 and 9, it can not be anything else than O; and as no rule is violated by inferring O in this case, it is O.

7. IA :—The conclusion is I. For by Rules 7 and 9, it can not be anything else than I; and as no rule is violated by inferring I in this case, it is I.

8. IE :—No conclusion follows. For by Rules 6 and 9, it can not be anything else than O; but as Rule 4 is violated by inferring O in this case, no conclusion follows.

9. OA :—The conclusion is O. For by Rules 6 and 9, it can not be anything else than O; and as no rule is violated by inferring O in this case, it is O.

That the conclusions proved above by the syllogistic rules are really valid, can be shown by the comparison of the diagrams, as in the case of the first and second figures.

The combinations AA, AI, EA, EI, IA, and OA yield, therefore, valid conclusions in the 3rd figure, and give rise to the following moods—AAI, AII, EAO, EIO, IAI, and OAO, technically called *Darapti*, *Datisi*, *Felapton*, *Ferison*, *Disamis*, and *Bokardo.*

From these valid moods we can generalize the following special rules of the third figure :—

(1) The minor premiss must be affirmative.

(2) The conclusion must be particular.

These two rules, which hold good in all the above-mentioned valid moods in the 3rd figure, may be thus proved by the general syllogistic rules. If the minor premiss be negative, the conclusion must be negative by Rule 6, and the major term, the predicate in the conclusion, will be distributed, which has not been distributed in the premisses, being the predicate in the major premiss, which must be affirmative by Rule 5. If the conclusion be universal, the minor term, the subject in the conclusion, will be distributed, which, being the predicate in the affirmative minor premiss, has not been distributed in the premisses.

§ 10. Valid Moods in the Fourth Figure.

1. AA :—The conclusion is I.

A. All A is B,
A. All B is C;
I. ∴ Some C is A.

2. AE :—The conclusion is E, and O follows from E by subalternation.

3. AI :—Invalid by Rule 3.

4. AO :—Invalid by Rule 3.

5. EA :—The valid conclusion is O.

6. EI :—The valid conclusion is O.

7. IA :—The valid conclusion is I.

8. IE :—Invalid by Rules 6 and 4.

9. OA :—Invalid by Rules 6 and 4.

In the 4th figure, the moods AAI, AEE, EAO, EIO and IAI, technically called *Bramantip*, *Camenes*, *Fesapo*, *Fresison*, and *Dimaris*, are valid. That these moods are really valid in the 4th figure can be proved by the comparison of the diagrams in the way in which we have proved the valid moods in the 1st and in the 2nd figure.

From these valid moods we can generalize the following special rules of the fourth figure:—

(1) If the major premiss be affirmative, the minor must be universal.

(2) If the minor premiss be affirmative, the conclusion must be particular.

(3) If either premiss be negative, the major premiss must be universal.

The student is required to prove these special rules by the general syllogistic rules.

§ 11. Questions and Exercises.

1. Define the following:—The major, minor, and middle terms, the major and minor premisses, the conclusion, syllogism, figure, mood, inference.

2. State the two axioms used in drawing inferences by the comparison of the diagrams.

3. Explain and illustrate the method of drawing inferences by the comparison of the diagrams.

4. Explain and illustrate the method of testing syllogisms by the comparison of the diagrams.

5. Define a syllogism, and show how the general syllogistic rules follow from its definition.

6. Prove as thoroughly as you can the following general syllogistic rules:—

(1) The middle term must be distributed at least once in the premisses.

(2) No term must be distributed in the conclusion which was not distributed in one of the premisses.

(3) If both the premisses be negative, nothing can be inferred.

(4) If one premiss be negative, the conclusion must be negative.

7. Explain fully the meaning of the terms 'figure' and 'mood.' How many figures are there? and how many moods? Give reasons for your answer.

8. Name the figure or figures in which the combination AA leads to a valid conclusion, giving reasons and concrete examples.

9. Name the figure or figures in which the combination AEE forms a valid mood, giving reasons and illustrations.

10. Give concrete examples of the following combinations of premisses in every figure, and draw the conclusions, if any, which follow from them, giving reasons:—AE, OA, IA, and IE.

11. Draw the conclusion, if any, which follows from each of the following combinations of premisses in any figure by the comparison of the diagrams:—AA, EA, AO, and EI.

12. Test by the comparison of the diagrams the following combinations in every figure:—AEA, IAA, AIA, EIE, AAA, EAE.

13. Prove the following general syllogistic rules:—

(1) If both the premisses be particular, nothing can be inferred.

(2) If one of the premisses be particular, the conclusion must be particular.

(3) To prove a negative conclusion one of the premisses must be negative.

(4) If the conclusion be affirmative, both the premisses must be affirmative.

(5) If the conclusion be universal, both the premisses must be universal.

(6) If both the premisses be affirmative, the conclusion must be affirmative.

14. Prove, by the general syllogistic rules, the following special rules :—

(1) In the first figure the major premiss must be universal.

(2) In the second figure the major premiss must be universal.

(3) In the third figure the minor premiss must be affirmative.

(4) In the fourth figure one of the premisses cannot be a particular negative.

(5) In the first figure the conclusion must have the quality of the major premiss and the quantity of the minor.

(6) In the second figure the conclusion must be negative and have the quantity of the minor premiss.

(7) In the third figure the conclusion must be particular and have the quality of the major premiss.

(8) In the fourth figure the conclusion cannot be an universal affirmative.

15. Name the figure or figures (1) in which A can be proved, (2) in which E can be proved, (3) in which I can be proved, and (4) in which O can be proved.

16. Name the moods which have A, E, I, and O respectively for their conclusions.

17. Give concrete examples of the moods AII, IAI, OAO, and EAO in those figures in which they are valid.

18. State and prove the special rules of the first figure, and determine by them the valid moods in that figure.

19. State and prove the special rules of the second figure, and determine by them the valid moods in that figure.

20. State and prove the special rules of the third figure, and determine by them the valid moods in that figure.

21. State and prove the special rules of the fourth figure, and determine by them the valid moods in that figure.

CHAPTER IV.

THE ARISTOTELIAN AND THE SCHOLASTIC METHODS OF DETERMINING VALID MOODS.

§ 1. Aristotle's *Dictum de omni et nullo*:—This celebrated Dictum is the supreme axiom or principle of syllogistic reasoning according to Aristotle and his followers, both ancient and modern. It is thus translated by Whately: "Whatever is predicated of a term distributed, whether affirmatively or negatively, may be predicated in like manner of anything contained in it." Mill states it as follows: "Whatever can be affirmed (or denied) of a class may be affirmed (or denied) of everything included in the class." The *Dictum* is self-evident, being an explicit statement of the *nature of a class*. A class is an indefinite number of individuals, possessing a common nature or certain attributes in common. Whatever possesses those attributes belongs to the class, or is included in it. Whatever does not possess them is not included. The very condition of a thing's belonging to the class is that it must possess the attributes which constitute the nature of the class. And the Dictum in its affirmative form simply states that what belongs to a higher class must belong to a lower, that is, to a class or to a thing included in the former, as otherwise it could not be so included; and in its negative form, it states that what does not belong to a class can not belong to any lower class or to any individual included in the former, as otherwise it could not be so included. In the proposition "All men are mortal," 'mortal' is affirmed of the class 'man,' and therefore it may be affirmed of any class, of any part of a class, or of any individual, such as 'all kings,' 'some beings,' or

'Socrates,' included in the higher class 'man.' In the proposition "No man is perfect," 'perfect' is denied of the class 'man,' and it may therefore be denied of any class, of any part of a class, or of any individual, such as 'all kings,' 'some beings,' or 'Socrates,' included in the higher class 'man.' These reasonings, stated fully, give rise to the following syllogisms:—(1) All men are mortal, all kings are men, therefore all kings are mortal; (2) all men are mortal, some beings are men, therefore some beings are mortal; (3) all men are mortal, Socrates is a man, therefore Socrates is mortal. And (1) no man is perfect, all kings are men, therefore no kings are perfect; (2) no man is perfect, some beings are men, therefore some beings are not perfect; (3) no man is perfect, Socrates is a man, therefore Socrates is not perfect.

§ 2. By applying the Dictum to the possible combinations of premisses we have given in the preceding chapter, it can be easily shown that only four (or six including the subalterns) are valid in the 1st figure, giving rise to the four moods we have already established. From the Dictum, we can easily deduce the two special rules of the 1st figure. According to the first clause of it, something must be affirmed or denied of a class distributively, that is, the major premiss must be universal, affirmative or negative. According to the last part of its second clause, something must be contained in the class, that is, the minor premiss must be affirmative. And these are the two special rules for the 1st figure. Applying the second of these two rules to the 16 combinations, we reject AE, AO, EE, EO, IE, IO, OE, and OO, and applying the first, we reject IA, II, OA, OI; and the remaining four AA, AI, EA, and EI, according to the first part of the second clause, give rise to the valid moods *Barbara*, *Darii*, *Celarent*, and *Ferio*.

§ 3. The Dictum is directly applicable to syllogisms in the 1st figure only, and can not be applied to any syllogism in the other figures. Hence Aristotle regarded the 1st figure as *perfect*, as the very type of syllogistic reasoning, and the other figures

as *imperfect*. He recognized only the first three figures, of which the first was considered to be the normal and standard mode of reasoning, and the other two as deviations from it, allowed for special purposes, as figures in Rhetoric are admissible deviations from the normal mode of expression; indeed, the word 'figure' as used in Logic has been borrowed from Rhetoric. The fourth figure is said to have been introduced by Galen, and is often called Galen's figure.

§ 4. Of Reduction:

Regarding all the figures except the first as imperfect, as having no principles or axioms by which to prove syllogisms in those figures with the same cogency as the *Dictum de omni et nullo* proves those in the first, Aristotle did not recognize any syllogism as valid unless it could be transformed into one in the perfect figure, and submitted to the test of his Dictum. This transformation of a syllogism in the second, third, or fourth figure into one in the first figure is technically called *Reduction*. Whether a particular syllogism in any imperfect figure is valid or not, is to be determined by its reduction to the first. If it can be so reduced, it is valid. If not, not. Aristotle determined entirely by this method the validity of syllogistic forms in the imperfect figures. Later logicians have, by the syllogistic rules, or by the special rules, or by other methods, first determined the valid moods in those figures, and then given directions for reducing them to the first, so that the Dictum may be ultimately applied to them. Whatever method is adopted, the valid moods in the other figures are the same as those we have obtained by the joint method of the comparison of the diagrams and the syllogistic rules. The valid moods in all the figures are given in the following mnemonic verses:—

Barbara, *Celarent*, *Darii*, *Ferioque*, prioris;
Cesare, *Camestres*, *Festino*, *Baroko*, secundæ;
Tertia, *Darapti*, *Disamis*, *Datisi*, *Felapton*,
Bokardo, *Ferison*, habet; quarta insuper addit
Bramantip, *Camenes*, *Dimaris*, *Fesapo*, *Fresison*.

These lines mean that there are four valid moods in the first figure, and four in the second, that the third figure contains six valid moods, and the fourth five. The three vowels in the name of each of the moods stand for the three propositions of the mood—the 1st for its major premiss, the 2nd for its minor premiss, and the 3rd for its conclusion. Thus the three vowels EAE in the mood *Celarent* signify that the major premiss is an E proposition, the minor an A proposition, and the conclusion an E proposition; and so with the rest.

There are two methods of reducing the imperfect moods, that is, the moods in the imperfect figures to the perfect; or rather of proving the truth of the conclusion of a mood in an imperfect figure by reduction to a perfect mood, that is, to a mood in the perfect figure:—(1) the one is called Direct or Ostensive Reduction, and (2) the other Indirect Reduction or *Reductio per deductionem ad impossibile* (*i.e.* Reduction by deduction to impossibility). In the first method the premisses of an imperfect mood are converted, obverted, contraposed, or transposed in order to form with them a mood in the first figure, having a conclusion which is the same as the original conclusion, or from which the original conclusion can be obtained by some process of immediate inference. In the second method, the truth of the conclusion of an imperfect mood is proved by showing, with the aid of the perfect moods and the rules of immediate inference by Opposition, that the contradictory of the conclusion is false.

§ 5. Ostensive Reduction:

The processes to be employed for reducing the imperfect moods by this method are indicated by certain letters contained in the names of the various moods. The initial letters B, C, D, F indicate that the imperfect moods are to be reduced to the perfect moods, having the same initial letters. The letter *s* means that the proposition signified by the vowel before it is to be converted *simply*. The letter *p* indicates that the proposition signified by the vowel before it is to be converted *by limitation* (*per accidens*). When *s* or *p* occurs after the conclusion

of an imperfect mood, *i.e.*, after the third vowel in its name, then its signification is to be applied to the conclusion of the new syllogism, that is, this conclusion must be converted simply in the case of *s* or by limitation in the case of *p* in order to obtain the conclusion of the imperfect mood. The letter *m* means that the premisses of the imperfect syllogism are to be transposed. The letter *k* means that the mood containing it was reduced by the older logicians by the Indirect method. The other letters (namely *l*, *r*, *n*, *t*) are entirely meaningless, and are introduced only for phonetic purposes to make up clearly sounding words. Thus C in *Camestres* means that it is to be reduced to *Celarent*; *m* that the premisses are to be transposed, that is, the major premiss of this is to become the minor of the new syllogism, and the minor the major premiss; the *s* after the minor premiss, that that premiss is to be converted simply; and the *s* after the conclusion or the third vowel, that the conclusion of the new syllogism in the mood *Celarent* is to be converted simply in order to obtain the original conclusion; while the consonants *t*, *r* are entirely non-significant.

I. Take, for example, the mood *Camestres* of the 2nd figure:—

(A)	All A is B	All metals are elements,
(E)	No C is B	No compounds are elements;
(E)	∴ No C is A	∴ No compounds are metals.

By converting simply the minor premiss, and transposing the premisses of this, we get the following new syllogism in the perfect mood *Celarent*:—

(E)	No B is C	No elements are compounds,
(A)	All A is B	All metals are elements;
(E)	∴ No A is C	∴ No metals are compounds.

The converse of the conclusion of the new syllogism is the same as the conclusion of the original syllogism.

II. Take the mood *Festino* of the 2nd figure—

(E)	No A is B	No men are perfect,
(I)	Some C is B	Some beings are perfect;
(O)	∴ Some C is not A	∴ Some beings are not men.

By converting simply the major premiss we get the following :—

(E)	No B is A	No perfect beings are men,
(I)	Some C is B	Some beings are perfect;
(O)	∴ Some C is not A	∴ Some beings are not men.

This is in the perfect mood *Ferio*. Its conclusion is the same as that of the original syllogism.

III. Take the mood *Darapti* of the 3rd figure—

(A)	All B is A	All men are rational,
(A)	All B is C	All men are imperfect;
(I)	∴ Some C is A	∴ Some imperfect beings are rational.

By converting *per accidens* the minor premiss we get the following :—

(A)	All B is A	All men are rational,
(I)	Some C is B	Some imperfect beings are men;
(I)	∴ Some C is A	∴ Some imperfect beings are rational.

This is in the perfect mood *Darii*. Its conclusion is the same as that of the original syllogism.

IV. Take the mood *Felapton* of the 3rd figure—

(E)	No B is A	No men are perfect,
(A)	All B is C	All men are rational;
(O)	∴ Some C is not A	∴ Some rational beings are not perfect.

By converting *per accidens* the minor premiss we get the following :—

(E)	No B is A	No men are perfect,
(I)	Some C is B	Some rational beings are men;
(O)	∴ Some C is not A	∴ Some rational beings are not perfect.

This is in the perfect mood *Ferio*. Its conclusion is the same as that of the original syllogism.

V. Take the mood *Bramantip* of the 4th figure—

(A)	All A is B	All men are imperfect,
(A)	All B is C	All imperfect things perish;
(I)	∴ Some C is A	∴ Some perishing things are men.

By transposing the premisses we get the following :—

(A)	All B is C	All imperfect things perish,
(A)	All A is B	All men are imperfect;
(A) ∴	All A is C	∴ All men perish.

This is a syllogism in the perfect mood *Barbara*. The converse of its conclusion is the same as the conclusion of the original syllogism.

VI. Take the mood *Dimaris* of the 4th figure—

(I)	Some A is B	Some men are wise,
(A)	All B is C	All wise beings are happy;
(I) ∴	Some C is A	∴ Some happy beings are men.

By transposing the premisses we get the following:

(A)	All B is C	All wise beings are happy,
(I)	Some A is B	Some men are wise;
(I) ∴	Some A is C	∴ Some men are happy.

This is a syllogism in the perfect mood *Darii*. The converse of its conclusion is the same as the conclusion of the original syllogism.

VII. Take the mood *Fresison* of the 4th figure—

(E)	No A is B	No man is perfect,
(I)	Some B is C	Some perfect beings are infallible;
(O) ∴	Some C is not A	∴ Some infallible beings are not men.

By converting simply the major and the minor premisses we get the following :—

(E)	No B is A	No perfect being is man,
(I)	Some C is B	Some infallible beings are perfect;
(O) ∴	Some C is not A	∴ Some infallible beings are not men.

This is in the perfect mood *Ferio*. The conclusion is the same as the original conclusion.

The directions given above for reduction are not sufficient for the two imperfect moods *Baroko* and *Bokardo*. The older logicians reduced them by the method to be next described, namely, Indirect Reduction. They may be, however, reduced to the first figure, by the method of Direct Reduction, thus :—

VIII. *Baroko* of the 2nd figure—

(A)	All A is B	All men are mortal,
(O)	Some C is not B	Some beings are not mortal;
(O) ∴	Some C is not A	∴ Some beings are not men.

By contraposing the major premiss, and obverting the minor premiss, we get the following syllogism :—

(E)	No not-B is A	No immortal being is man,
(I)	Some C is not-B	Some beings are immortal;
(O) ∴	Some C is not A	∴ Some beings are not men.

This is a syllogism in the perfect mood *Ferio*, of which 'A' and 'C' are the major and minor terms, and 'not-B' the middle term.

IX. *Bokardo* of the 3rd figure—

(O)	Some B is not A	Some men are not wise,
(A)	All B is C	All men are rational;
(O) ∴	Some C is not A	∴ Some rational beings are not wise.

By contraposing the major premiss, and transposing the premisses, we get the following syllogism :—

(A)	All B is C	All men are rational,
(I)	Some not-A is B	Some not-wise are men;
(I) ∴	Some not-A is C	∴ Some not-wise are rational.

This is a syllogism in the perfect mood *Darii*, of which 'C' and 'not-A' are the major and minor terms, and 'B' the middle term. By converting simply the conclusion of the new syllogism and then obverting the converse, we can easily obtain the conclusion of the original syllogism.

The processes employed for reducing them will be sufficiently indicated if *Baroko* and *Bokardo* be called *Facoko* and *Doclamosk* respectively, *c* signifying that the proposition signified by the vowel before it is to be contraposed, *k* that the proposition is to be obverted, and *s* as usual, that the proposition is to be simply converted.

§ 6. Indirect Reduction, or, *Reductio per deductionem ad impossibile*.

I. *Baroko* of the second figure may be thus reduced by this method :—

(A) All A is B,
(O) Some C is not B;
(O) ∴ Some C is not A.

The conclusion of this syllogism is true if the premisses be true. If the conclusion 'Some C is not A' be not true, then its contradictory 'All C is A' must be true by Opposition, because of two contradictory propositions one must be true. Then combining this with the major premiss of the given syllogism, we have the following new syllogism in the perfect mood *Barbara* :—

(A) All A is B,
(A) All C is A;
(A) All C is B.

If the conclusion of this syllogism be true, its contradictory 'Some C is not B' must be false by Opposition; because of two contradictory propositions one must be false. But the latter is the minor premiss of the original syllogism, and is therefore true by supposition. Hence its contradictory, the conclusion of the new syllogism, must be false; and the falsity must be due either to the process of reasoning or to the premisses. The falsity can not be due to the process of reasoning, for the new syllogism is in the perfect mood *Barbara;* it must therefore be due to the premisses. It can not be due to the major premiss, which is also the major premiss of the original syllogism, and is therefore true by supposition: hence it must be due to the minor premiss 'All C is A,' that is, this premiss must be false, and its contradictory 'Some C is not A,' the conclusion of the original syllogism, is therefore true.

II. *Bokardo* of the 3rd figure may be thus reduced by this method :—

(O) Some B is not A,
(A) All B is C;
(O) ∴ Some C is not A.

The conclusion of this syllogism is true, if the premisses be true. If the conclusion be not true, its contradictory 'All C is A' must be true by Opposition. Then taking this as a major premiss, and the minor premiss of the original syllogism as a minor premiss, we can form the following new syllogism in the perfect mood *Barbara*:—

(A) All C is A,
(A) All B is C;
(A) ∴ All B is A.

If the conclusion 'All B is A' be true, then its contradictory 'Some B is not A' must be false by Opposition; but this is not possible, as the latter is the major premiss of the original syllogism, and therefore true by supposition. Hence the former 'All B is A' must be false; and the falsity not being due to the reasoning process which is in the perfect mood *Barbara*, nor to the minor premiss 'All B is C' of the new syllogism, which is also the minor premiss of the original syllogism, and therefore true by supposition, it must be due to the falsity of the major premiss 'All C is A.' This proposition being false, its contradictory 'Some C is not A,' the conclusion of the original syllogism, is true.

The initial letter *B* of these two moods signifies that the new syllogism which arises in the process of reduction is in the mood *Barbara*, and the letter *k* indicates that the older logicians reduced them by the Indirect method.

The Indirect method of Reduction is also applicable to the other imperfect moods.

III. Take, for example, *Cesare* of the 2nd figure—

(E) No A is B,
(A) All C is B;
(E) ∴ No C is A.

If this conclusion be not true, its contradictory 'Some C is A' must be true by Opposition. We can now form the following new syllogism in the perfect mood *Ferio*—

(E) No A is B,
(I) Some C is A;
(O) ∴ Some C is not B.

If this conclusion be true, its contradictory 'All C is B' must be false. But this is not possible, as the proposition 'All C is B' is the minor premiss of the original syllogism, and therefore true by supposition. Hence the conclusion of the new syllogism is not true; and its falsity not being due to the reasoning process, nor to the major premiss of the syllogism, must be due to the falsity of the minor premiss 'Some C is A.' Hence this proposition is false, and its contradictory 'No C is A,' the conclusion of the original syllogism, is true.

IV. Take the mood *Darapti* of the 3rd figure—

(A) All B is A,
(A) All B is C;
(I) ∴ Some C is A.

If this conclusion be not true, its contradictory 'No C is A' must be true. With this as a major premiss, and the minor premiss of the original syllogism as a minor premiss, we can form the following new syllogism in the perfect mood *Celarent*—

(E) No C is A,
(A) All B is C;
(E) ∴ No B is A.

If this conclusion be true, its contrary 'All B is A' must be false by Opposition, because two contrary propositions can not both be true, and one must be false. But 'All B is A' being the major premiss of the original syllogism can not be false; hence 'No B is A,' the conclusion of the new syllogism, can not be true and must be false, the falsity being due, as in the preceding cases, to the major premiss 'No C is A' being false. This proposition being false, its contradictory 'Some C is A,' the conclusion of the original syllogism, must be true.

§ 7. Exercises.

1. What is Reduction? Is it necessary? Define Direct and Indirect Reduction, and distinguish them from each other.

2. Reduce by the Direct method the following moods:—Cesare, Disamis, Datisi, Ferison, Bramantip, Camenes, and Fesapo.

3. Reduce the following moods by the Indirect method:—Camestres, Felapton, Bramantip, Festino, Camenes, Dimaris, and Disamis.

4. Reduce both by the Direct and by the Indirect method the two moods Baroko and Bokardo.

5. Show by the Aristotelian method that the moods AAA, EAA, AII, and AEA are invalid in the second figure.

6. Find by the same method the conclusion, if any, to which the following combinations lead in the imperfect figures:—AA, AE, EA, OA, AO, and EI.

7. Show by the same method that the moods AAA, EAE, AEE are invalid in the third figure.

8. Determine by the same method the valid moods in the second figure.

9. Give concrete examples of the following moods, and reduce them both by the Direct and by the Indirect method:—Bramantip, Disamis, Baroko, Fesapo, and Bokardo.

10. Reduce the following pairs of premisses to the first figure and draw the conclusion, if any, which follows from each pair:—

(i) No X is Y, all Y is Z. (iii) All Y is X, all Y is Z.
(ii) No X is Y, all Z is Y. (iv) No Y is X, all Y is Z.

11. Test the following inferences by the method of Diagrams and also by the Aristotelian and scholastic methods.

(i) No A is B; no C is not-B; therefore all C is not-A.
(ii) All A is B; all C is not-B; therefore no C is A.
(iii) No not-B is C; all not-B is A; therefore some C is not-A.
(iv) None but material bodies gravitate; air gravitates: therefore air is a material body.
(v) Plants alone have flowers; zoophytes have no flowers: therefore they are not plants.

CHAPTER V.

The Various Kinds of Syllogisms.

§ 1. A Syllogism consists of two premisses and the conclusion which follows from them. It is evident that the two premisses of a syllogism may differ in Quality, Quantity, Relation, or Modality. The various kinds or divisions of syllogisms are founded upon the modifications of these general characters of their premisses. We have seen in a previous chapter that the division into Moods is founded upon the difference in Quantity and Quality of the two premisses. The division of syllogisms into Pure and Mixed is founded upon the difference in Relation of the premisses. The division into (1) Necessary, (2) Assertory, and (3) Probable is founded upon the difference in Modality of the premisses. The various kinds or divisions may be shown thus in a tabular view :—

SYLLOGISMS.	Quality and Quantity ...	Moods.
	Relation ...	1. Pure. (Both the premisses of the same relation.) 2. Mixed. (Premisses of different relations, *e.g.*, one categorical, and the other hypothetical, &c.)
	Modality ...	1. Necessary. 2. Assertory. 3. Probable.

The two classes of Pure and Mixed syllogisms, founded on the difference in Relation of the premisses, are thus subdivided. If the premisses of a pure syllogism are both categorical or both hypothetical, the pure syllogism is Categorical or Hypothetical. If a mixed syllogism has one premiss categorical and the other hypothetical, or one premiss categorical and the other disjunctive, or lastly, one conjunctive and the other disjunctive, it is called (1) Hypothetical-categorical, (2) Disjunctive-categorical, or (3) Conjunctive-disjunctive. By a conjunctive proposition is meant a compound proposition consisting of two categoricals or two hypotheticals connected by the two conjunctions 'neither ... nor' or 'as well as.'

The subdivisions may be shown in a tabular view :—

SYLLOGISMS.

- I. Pure ...
  1. Categorical, consisting of two categorical premisses.
  2. Hypothetical, consisting of two hypothetical premisses.
- II. Mixed.
  1. Hypothetical-categorical, consisting of one premiss hypothetical and the other categorical.
  2. Disjunctive-categorical, consisting of one premiss disjunctive and the other categorical.
  3. Conjunctive-disjunctive, consisting of one premiss conjunctive and the other disjunctive.

§ 2. I.—Of Pure Syllogisms.

The general syllogistic rules and the special rules which we have given in a previous chapter are applicable to hypothetical, as well as to categorical, syllogisms. Of the latter we have given numerous examples. We shall now give some examples of the former. In applying the general and the special rules to pure hypothetical syllogisms, we must remember (1) that the antecedent of a hypothetical proposition corresponds to the subject, and the consequent to the predicate in the corresponding categorical proposition; (2) that the quantity of a hypothetical proposition is the quantity of its antecedent, and is expressed

by such phrases as 'in all cases' and 'in some cases' or 'in one case at least,' the former denoting universal and the latter particular quantity; (3) that the quality of a hypothetical proposition is the quality of its consequent; (4) that the rules for the distribution of terms are the same as in categorical propositions, *i.e.*, the antecedent must be distributed in hypothetical propositions of the form A or E, and the consequent in hypothetical propositions of the form E or O. We shall give the following typical examples of Pure Hypothetical Syllogisms, and change them at the same time into the corresponding Categoricals :—

FIRST FIGURE.

I.—Barbara :—

A. In all cases, if B is, C is ... (major premiss),
A. In all cases, if A is, B is ... (minor premiss);
A. ∴ In all cases, if A is, C is ... (conclusion).

Changed into the corresponding categorical:

Every case of the existence of B is a case of the existence of C,
Every case of the existence of A is a case of the existence of B;
∴ Every case of the existence of A is a case of the existence of C.

II.—Celarent :—

E. In all cases, if B is, C is not ... (major premiss),
A. In all cases, if A is, B is ... (minor premiss);
E. ∴ In all cases, if A is, C is not ... (conclusion).

Changed into the corresponding categorical:

No case of the existence of B is a case of the existence of C,
Every case of the existence of A is a case of the existence of B;
∴ No case of the existence of A is a case of the existence of C.

III.—Darii :—

A. In all cases, if B is, C is ... (major premiss),
I. In some cases, if A is, B is ... (minor premiss);
I. ∴ In some cases, if A is, C is ... (conclusion).

Changed into the corresponding categorical:

Every case of the existence of B is a case of the existence of C,
Some cases of the existence of A are cases of the existence of B;
∴ Some cases of the existence of A are cases of the existence of C.

SECOND FIGURE.

IV.—Cesare:—

E. In all cases, if C is, B is not ... (major premiss),
A. In all cases, if A is, B is ... (minor premiss);
E. ∴ In all cases, if A is, C is not ... (conclusion).

Changed into the corresponding categorical:

No case of the existence of C is a case of the existence of B,
Every case of the existence of A is a case of the existence of B;
∴ No case of the existence of A is a case of the existence of C.

V.—Camestres:—

A. In all cases, if A is, B is ... (major premiss),
E. In all cases, if C is, B is not ... (minor premiss);
E. ∴ In all cases, if C is, A is not ... (conclusion).

THIRD FIGURE.

VI.—Darapti:—

A. In all cases, if B is, C is ... (major premiss),
A. In all cases, if B is, A is ... (minor premiss);
I. ∴ In some cases, if A is, C is ... (conclusion).

Similar examples may be given of the fourth figure, and also of the other moods of the first three figures.

§ 3. II.—Of Mixed Syllogisms.

We have seen that there are at least three subdivisions, namely, (1) Hypothetical-categorical, (2) Disjunctive-categorical, (3) Conjunctive-disjunctive. We shall take these in order—

1. Of Hypothetical-categorical Syllogisms.

A syllogism of this subdivision consists of a hypothetical major and a categorical minor premiss, the conclusion being categorical. The rules of inference are as follows :—

A. When the hypothetical major premiss is affirmative.

(1) If you affirm the antecedent, you may affirm the consequent of a hypothetical premiss, but not conversely, that is, it is not allowed to affirm the antecedent on affirming the consequent.

(2) If you deny the consequent, you may deny the antecedent of a hypothetical premiss, but not conversely, that is, it is not allowed to deny the consequent on denying the antecedent.

B. When the hypothetical major premiss is negative.

(3) If you affirm the antecedent you may deny the consequent, but not conversely, that is, it is not allowed to affirm the antecedent on denying the consequent.

(4) If you affirm the consequent you may deny the antecedent, but not conversely, that is, it is not allowed to affirm the consequent on denying the antecedent.

These rules follow from the meaning of an hypothetical proposition (see p. 130). The second part of the first rule follows from the fact that the consequent may depend upon other antecedents as well as upon that antecedent, and that therefore the existence or affirmation of the consequent does not necessarily imply the affirmation of that particular antecedent, but of some one of them, and this one may not be the antecedent in question. The second part of the second rule follows from the same fact, for the consequent depending, as it may, on other antecedents as well, may exist while the particular antecedent is absent; and therefore the denial of the consequent does not follow from the denial of the antecedent. For example, in the proposition "If a person be attacked with cholera, he will die," —assuming this to be true—it does not follow that, if he be not attacked with cholera, he will not die; for he may die of consumption, fever, or some other disease. Nor does it follow that if he dies, he must have been attacked with cholera, for he may die of other diseases. All that is really meant by the proposition in question is that if he gets cholera, he is sure to die; if the antecedent is present, the consequent must be present, and that if he does not die, he has not had cholera, *i.e.* if the consequent does not occur, the antecedent can not have occurred.

We shall give some typical examples of Hypothetical-categorical syllogisms, and change them at the same time into

the corresponding categoricals, in order to show that, when thus changed, they conform to the fundamental rules and axioms of categorical syllogisms :—

1. In all cases, if A is, B is,
A is ;
∴ B is.

Here the hypothetical major premiss is affirmative and the conclusion follows according to the first rule given above.

It may be thus changed into a categorical :—

A. Every case of the existence of A is a case of the existence of B,
A. This is a case of the existence of A ;
A. ∴ This is a case of the existence of B.

The syllogism is in the mood *Barbara.*

A Hypothetical-categorical syllogism may be also changed into a pure hypothetical syllogism ; for the meaning of the minor proposition 'A is' is, that 'if this case is, A is.' By substituting this hypothetical minor premiss for the categorical, we get a pure hypothetical syllogism in the mood *Barbara*, thus :—

In all cases, if A is, B is ... ... (major premiss),
If this case is, A is ... ... (minor premiss);
∴ If this case is, B is ... ... (conclusion).

The conclusion when changed into the categorical form is 'B is.'

The converse of the first rule does not lead to a valid syllogism—

In all cases, if A is, B is,
B is;
∴ A is.

This inference is not valid ; and its invalidity can be shown by changing it into the corresponding categorical, when it will be seen that the latter violates some of the syllogistic rules, thus :—

Every case of the existence of A is a case of the existence of B,
This is a case of the existence of B,

From these two premisses no conclusion follows, because the middle term 'a case of the existence of B' is not distributed in either premiss.

2. In all cases, if A is, B is not,
A is;
∴ B is not.

Here the hypothetical major premiss is negative and the conclusion follows according to the third rule given above. If the major premiss is taken as affirmative, the same conclusion follows according to the first rule given above.

It may be thus changed into a categorical:

E. No case of the existence of A is a case of the existence of B,
A. This is a case of the existence of A;
E. ∴ This is not a case of the existence of B.

This is a syllogism in the mood *Celarent* of the 1st figure.

It may also be changed into a pure hypothetical syllogism, thus:

E. In all cases, if A is, B is not ... ... (major premiss),
A. If this case is, A is ... ... (minor premiss);
E. ∴ If this case is, B is not ... ... (conclusion).

Similarly, hypothetical-categorical syllogisms corresponding to *Darii* and *Ferio* may be easily formed by making the minor premiss particular.

3. In all cases, if A is, B is,
B is not;
∴ A is not.

Here the hypothetical major premiss is affirmative and the conclusion follows according to the second rule given above. It may be thus changed into *Camestres* in the 2nd figure:

Every case of the existence of A is a case of the existence of B ... ... (major premiss),

This is not a case of the existence of B ... (minor premiss);
∴ This is not a case of the existence of A ... (conclusion).

In all cases, if A is, B is ... ... (major premiss),
If this case is, B is not ... ... (minor premiss);
∴ If this case is, A is not ... ... (conclusion).

The converse of the second rule does not lead to a valid syllogism. That no inference can be drawn conversely may be easily shown thus:—

In all cases, if A is, B is,
A is not;
∴ B is not.

This inference can not be drawn, as will be evident, when the syllogism is changed into the corresponding categorical:

Every case of the existence of A is a case of the existence of B,
This is not a case of the existence of A;
∴ This is not a case of the existence of B.

Here the major term 'a case of the existence of B' is distributed in the conclusion, while it is not distributed in the premiss.

4. In all cases, if A is, B is not,
B is;
∴ A is not.

Here the hypothetical premiss is negative and the conclusion is drawn according to the fourth rule given above If the major premiss is taken as affirmative, the same conclusion follows according to the second rule given above.

It can be easily changed into *Cesare*:—

E. No case of the existence of A is a case of the existence of B,
A. This is a case of the existence of B;
E. ∴ This is not a case of the existence of A.

In all cases, if A is, B is not ... ... (major premiss),
If this case is, B is ... ... (minor premiss);
If this case is, A is not ... ... (conclusion).

To the typical forms given above may be added the following modifications of them:

5. In all cases, if A is not, B is,
A is not;
∴ B is.

It corresponds to the 1st example given above.

6. In all cases, if A is not, B is not,
A is not;
∴ B is not.

It corresponds to the 2nd example given above.

7. In all cases, if A is not, B is,
B is not;
∴ A is.

It corresponds to the 3rd example given above.

8. In all cases, if A is not, B is not,
B is;
∴ A is.

It corresponds to the 4th example given above.

§ 4. 2. Of Disjunctive-categorical Syllogisms.

The next subdivision under Mixed Syllogisms is that of Disjunctive-categorical Syllogisms. In the *wider sense* a syllogism of this subdivision consists of a disjunctive and a categorical premiss, and may occur in all figures.

In the First Figure, *Barbara:*

M is either A or B ... (major premiss),
C is M ... ... (minor premiss);
∴ C is either A or B ... (conclusion).

In the Second Figure, *Camestres:*

A is either M or N ... (major premiss),

C is neither M nor N ... (minor premiss);
∴ C is not A ... ... (conclusion).

In the Third Figure, *Darapti:*

M is either A or B ... (major premiss),
M is C ... ... (minor premiss);
∴ Some C is either A or B (conclusion).

In the Fourth Figure, *Bramantip:*

A is M ... ... (major premiss),
M is either B or C ... (minor premiss);
∴ Something which is either B or C is A ... (conclusion).

In the *stricter sense* Disjunctive-categorical Syllogisms consist of the following two forms only :—

1. A is either B or C,
A is not B;
∴ A is C.

2. A is either B or C,
A is not C;
∴ A is B.

To these two some logicians add the following two forms:

3. A is either B or C,
A is B;
∴ A is not C.

4. A is either B or C,
A is C;
∴ A is not B.

Of these four forms Mill admits only the first two as valid, while Ueberweg regards all of them as equally valid. We have already referred to the difference of opinion among logicians on this subject. Mill regards the disjunctive proposition 'A is either B or C' as equivalent to one or other of the following two hypotheticals :—(1) 'If A is not B, A is C,' and (2) 'If A is not C, A is B,' and accepts accordingly the first two only of the above-mentioned four forms, while Ueberweg regards the disjunc-

tive as equivalent to one or other of the following two hypotheticals as well :—(1) 'If A is B, A is not C,' and (2) 'If A is C, A is not B,' and thus accepts all the forms.

A Disjunctive-categorical may be easily changed into a Hypothetical-categorical syllogism; and we have seen that the latter may be changed into a pure hypothetical or into a pure categorical. Thus the first may ultimately be obtained in the categorical form, and tested by the canons and rules applicable to that form, thus:—

A is either B or C ... (major premiss),
A is not B ... ... (minor premiss);
∴ A is C ... ... (conclusion).

By change of Relation we obtain from the disjunctive major the following hypothetical,—'If A is not B, A is C.' This with the other two propositions will give a hypothetical-categorical syllogism which can be easily changed into a pure one in the mood *Barbara:*

In all cases, if A is not-B, A is C,
If this case is, A is not-B;
∴ If this case is, A is C.
} Hypothetical.

Every case of A being not-B, is a case of A being C,
This is a case of A being not-B;
∴ This is a case of A being C.
} Categorical.

Similarly the other disjunctive-categorical forms also may be ultimately changed into the corresponding categorical forms.

§ 5. 3. Of Conjunctive-Disjunctive Syllogisms, or the Dilemma.

The *Dilemma* is a conjunctive-disjunctive Syllogism, that is, a mixed syllogism having a conjunctive and a disjunctive premiss. A *conjunctive* proposition is a compound proposition consisting of two categoricals or two hypotheticals connected by the conjunctions, 'neither—nor' or 'as well as.' When the two constituent propositions are connected by 'neither—nor,' the con-

junctive proposition is called *Remotive;* and when they are connected by 'as well as,' it is called *Copulative.* There is great difference of opinion as to the true nature and forms of the Dilemma. Ueberweg's view, which appears to be the best, is given below. In the appendix will be found the views of other Logicians.

The Dilemma, Trilemma, Polylemma.

"In these inferences or arguments, it is shown that *whichever of the members of the disjunction be true,* the same conclusion results (that the opponent, whichever of the different possible cases he may choose, must find himself in every case forced to the same conclusion)[1]."

"The Dilemma, in the *stricter and special sense,* is an inference of the second figure, with a hypothetico-disjunctive premiss (which is sometimes major and sometimes minor premiss), and with a remotive premiss. In the *wider sense* of the term, inferences with a categorico-disjunctive premiss, and inferences in the first figure with a disjunctive and a copulative or remotive premiss, are also attributed to it. The like holds good of the Trilemma, Tetralemma, and Polylemma[2]."

In the *wider sense* the Dilemma is a conjunctive-disjunctive Syllogism of the second or the first figure, in which the conclusion is the same, whichever of the alternatives in the disjunctive premiss be true. In the *stricter sense,* the Dilemma is a conjunctive-disjunctive Syllogism of the second figure, in which the conjunctive premiss is remotive and the disjunctive premiss hypothetical, and in which the conclusion is the same, whichever of the alternatives in the disjunctive premiss be true. The essential conditions of a Dilemma in the *wider sense* are (1) that one premiss must be *disjunctive* and the other premiss *conjunctive,* (2) that the reasoning must be in the *second* or in the *first figure,* and (3) that *the same conclusion must follow,* whichever of the alternatives in the disjunctive premiss be true. The essential conditions of a Dilemma in the *stricter sense* are (1) that

[1] Ueberweg's *Logic,* p. 455. [2] *Ibid.* p. 457.

one premiss must be a *disjunctive proposition of the hypothetical form* such as "If A is, either B or C is," and the other premiss *a remotive proposition*, (2) that the reasoning must be in the *second figure*, and (3) that *the same conclusion must follow*, whichever of the alternatives in the disjunctive premiss be true. *The most essential condition of a dilemma is that the same conclusion must follow, whichever of the alternatives in the disjunctive premiss be true.* In the Trilemma, Tetralemma and Polylemma, the disjunctive premiss presents three, four, and more than four, alternatives, respectively, and the conclusion is the same, whichever of the alternatives be true.

Examples of the Dilemma.

I. Symbolical Examples.

1. A is either B or C,
D is neither B nor C;
∴ D is not A.

This is a dilemma in the *wider sense*. It may be analysed as follows:—

The *disjunctive* major premiss is equivalent to—

(1) A is B,
or (2) A is C, and the *remotive* minor premiss is equivalent to—
(1) D is not B,
and (2) D is not C.

From (1) of both the premisses

A is B,
D is not B;
∴ D is not A;

and from (2) of both—

A is C,
D is not C;
∴ D is not A.

Thus whichever of the two alternatives in the disjunctive premiss be true, the conclusion is the same (D is not A) as required by the definition of the Dilemma.

2. If A is, as well as if B is, C is,
   If D is, either A or B is;
   ∴ If D is, C is.

This is a Dilemma in the *wider sense.*

It may be thus analysed:—

The *copulative* major premiss is equivalent to—

(1) If A is, C is,
and (2) If B is, C is, and the *disjunctive* minor premiss is equivalent to—

(1) If D is, A is,
or (2) If D is, B is.

From (1) of both the premisses

If A is, C is,
If D is, A is;
∴ If D is, C is.

From (2) of both—

If B is, C is,
If D is, B is;
∴ If D is, C is.

Thus in either case, that is, whichever of the two alternatives in the disjunctive premiss be true, the conclusion is the same (If D is, C is) as required by the definition of the Dilemma.

3. If A is, either B or C is,
   Neither B nor C is;
   ∴ A is not.

This is a dilemma in the *stricter sense.*

It may be analysed as follows:—

The *disjunctive* major premiss is equivalent to—

(1) If A is, B is,
or (2) If A is, C is,

and the *remotive* minor premiss is equivalent to—

(1) B is not,
and (2) C is not.

From (1) of both the premisses,

If A is, B is,
B is not;
∴ A is not.

From (2) of both—

If A is, C is,
C is not;
∴ A is not.

The conclusion is the same, 'A is not,' whichever of the two alternatives in the disjunctive premiss be true.

II. Concrete Examples.

1. Electricity is either a form of matter or a mode of motion; Mind is neither a form of matter nor a mode of motion; therefore Mind is not electricity. (It corresponds to the first symbolical example given above.)

2. If this substance is a plant, as well as if it is an animal, it has life; if it has organic structure, it is either a plant or an animal; therefore, if it has organic structure, it has life. (It corresponds to the second symbolical example given above.)

3. If the water in this tube is heated, either its temperature rises or its volume increases; its temperature is not rising nor is its volume increasing; therefore, the water in this tube is not heated. (It corresponds to the third symbolical example given above.)

In testing a Dilemmatic argument the following method should be followed:—

(1) Each premiss should be analysed into the constituent propositions.

(2) The constituent propositions should be combined to form the constituent Syllogisms of the argument, and each Syllogism tested by its rules.

(3) It should be noticed whether the conclusion of each Syllogism is the same. If the conclusion is not the same, the argument is not a Dilemma.

Dilemmatic arguments generally contain *material* fallacies, i.e. consist of false premisses. For instance, the disjunctive premiss may be false, the opposition between its alternative members not being thorough-going and valid; the consequent in the hypothetical-disjunctive premiss may not follow from the antecedent; or the conjunctive premiss may be false. *Material* fallacies in Dilemmatic as well as in other arguments can not be detected without a competent knowledge of the subject-matter of the arguments.

Exercises.

Test the following arguments:—

(1) Hydrogen is either a metal or a non-metal; a compound substance is neither a metal nor a non-metal; therefore Hydrogen is not a compound substance.

(2) If this substance is an organism, it is either a plant or an animal; if this substance is a crystal, it is neither a plant nor an animal; therefore, if this substance is a crystal, it is not an organism.

(3) If a sensation is a bodily state, it is a molecular change either in the brain or in the organ of sense; a sensation is not a molecular change either in the brain or in the organ of sense; therefore, a sensation is not a bodily state.

(4) If A is, neither B nor C is; if D is, either B or C is: therefore, if D is, A is not.

(5) If A is, either B or C is; if D is, neither B nor C is: therefore, if D is, A is not.

(6) Neither if A is nor if B is, is C; if D is, either A or B is: therefore, if D is, C is not.

(7) If matter exists, it is either an impression or an idea; it is neither an impression nor an idea; therefore, it does not exist.

(8) "If virtue were a habit worth acquiring, it must insure either power, or wealth, or honour, or pleasure; but virtue insures none of these; therefore, virtue is not a habit worth attaining."

(9) A phenomenon is either an impression or an idea; matter is neither an impression nor an idea; therefore, matter is not a phenomenon.

(10) A is neither B nor C; D is either B or C: therefore, D is not A.

(11) If A is, neither B nor C is; either B or C is: therefore, A is not.

(12) A as well as B is C; D is either A or B: therefore, D is C.

(13) Neither A nor B is C; D is either A or B: therefore, D is not C.

(14) Neither if A is nor if B is, is C; now either A or B is: therefore, C is not.

(15) If the human soul is specially created as well as if it is evolved from the animal soul, it is not eternal; it is either specially created or evolved from the animal soul: therefore, it is not eternal.

§ 6. Exercises.

Test the following arguments:—

(1) If the sun shines, it will be a brilliant day; if it is not foggy or cloudy, the sun will shine; therefore, if it is not foggy or cloudy, it will be a brilliant day.

(2) If the temperature rises, the barometer will fall; if the barometer falls, the weather will not be fine; therefore, if the temperature rises, the weather will not be fine.

(3) If a gas is subjected to a higher pressure, its volume diminishes; if its volume diminishes, its density increases; therefore, if a gas is subjected to a higher pressure, its density increases.

(4) If the earth did not rotate, there would be no alternation of day and night; there is alternation of day and night; therefore the earth does rotate.

(5) Without light and heat, no plants could grow; without plants no animals could live; man, being an animal, could not, therefore, live without light and heat.

(6) An organized being is either an animal or a plant: this substance is neither; therefore it is not an organized being.

(7) If a substance has inertia, it has gravity; if it does not resist, it has no inertia; therefore, if a substance does not resist, it has no gravity.

(8) If a substance gravitates, it has inertia; if a substance has the power of resistance, it has inertia; therefore if a substance gravitates, it has the power of resistance.

(9) If a solid is heated, it becomes a liquid; if a liquid is heated, it becomes a gas: therefore if a solid is heated, it becomes a gas.

(10) If A is not, B is not; if B is not, C is not: therefore if A is not C is not.

(11) An igneous rock is either volcanic or plutonic; trap is a kind of igneous rock: therefore it is either volcanic or plutonic.

(12) A material body is either organic or inorganic; a crystal is not organic: therefore it is inorganic.

(13) If water is heated, either its bulk increases, or its temperature rises, or it passes into vapour; neither of these changes is happening to the water in this flask: therefore it is not heated.

(14) All existences are either mental or material; nothing is neither mental nor material: therefore nothing is not an existence.

(15) A liquid as well as a gas is expanded by heat; a fluid is either a gas or a liquid: therefore a fluid is expanded by heat.

(16) If the motion of a body is impeded, heat is produced; if heat is produced, the body will either rise in temperature or increase in bulk, or pass into a different state; therefore, if the motion of a body is impeded, the body will either rise in temperature, or increase in bulk, or pass into a different state.

(17) If every notion is derived from sensation or reflection, the notion of extension is also so derived. But it cannot be so derived. Therefore every notion is not derived from sensation or reflection.—*Reid's Inquiry.*

(18) If Nature had given us nothing more than sensations corresponding to the impressions made by the objects upon the body, we should not in that case have been percipient beings. But we are percipient beings; therefore Nature has given us more than the sensations.—*Reid.*

(19) Body and spirit, cause and effect, time and space, to which we were wont to ascribe an existence independent of our thought, are all turned out of existence by this short dilemma. Either these things are ideas of sensation or reflection or they are not: if they are ideas of sensation or reflection, they can have no existence but when we are conscious of them; if they are not ideas of sensation or reflection, they are words without any meaning.—*Reid.*

§ 7. Of Enthymemes[1].

An Enthymeme is an abridged syllogism, that is, a syllogism, one of whose premisses is not expressed in language. For example, 'gold is an element, because it is a metal'; here we have a syllogism apparently consisting of two propositions, but really of three, including the major premiss, which is suppressed, and which must be as follows:—'all metals are elements.' Without this, the conclusion 'gold is an element' can not be drawn from the single premiss 'gold is a metal.' Though the major premiss is not expressed in language, it is contended that it must have been present in thought to complete or constitute the act of reasoning. Similarly, the minor premiss may be sometimes suppressed. For example, 'all men are fallible, and therefore kings or philosophers are fallible.' Here the minor premiss 'kings or philosophers are men' is understood. Sometimes even the conclusion may be suppressed, and hinted at by the expression of the two premisses, or, in rare cases, of one only. This often happens in conversation between educated persons on delicate subjects. For example, happening to talk about

[1] The word *enthymeme* originally meant a syllogism with probable premisses. This is the sense in which Aristotle used it. It came afterwards to mean a syllogism which was imperfect not from its premisses being probable, but from one of them being suppressed, and in this sense the word is now used in Logic.

a particular institution which has been much praised and declared as *perfect*, an opponent might, in reply, simply say that 'everything human is imperfect,' or that 'everything is liable to change and decay': here nothing but the major premiss is expressed, and it is of course implied that 'the institution in question is human' (minor premiss), and that 'it is, therefore, not perfect' (conclusion).

§ 8. Exercises.

1. *To supply the suppressed premiss of an Enthymeme.* (1) Note the subject and the predicate in the conclusion which are the minor and the major term, respectively, of the syllogism, and then see whether the premiss to be supplied is the major or the minor premiss. (2) If it be the major premiss, form such a proposition with the major and the middle term as will make the conclusion valid. (3) If it be the minor premiss, form such a proposition with the minor and the middle term as will make the conclusion valid. Examples:—(1) "All metals are elements, because they can not be decomposed." In this the subject and the predicate in the conclusion are respectively 'all metals' and 'elements,' and these two are, therefore, the minor and the major term, respectively. The given premiss contains the minor term 'metals,' and is, therefore, the minor premiss. The premiss suppressed is, therefore, the major premiss, and is the proposition 'all substances that can not be decomposed are elements.' (2) "Small-pox has a cause, because every phenomenon has a cause." Here 'small-pox' is the minor term, 'has a cause' the major term, and 'phenomenon' the middle term. The premiss expressed containing the major term 'has a cause,' is the major premiss. The premiss suppressed is, therefore, the minor premiss, and is the proposition 'small-pox is a phenomenon.'

2. *To find premisses for a given conclusion.* In finding premisses for a given conclusion, note the subject and the predicate in the conclusion, which must be the minor and the major term, respectively, of the required syllogism. If the conclusion be negative, find such a middle term as will form with the predicate an E proposition, and with the subject an A or I proposition. If the conclusion be affirmative, find such a middle term as will form with the predicate an A proposition, and with the subject an A or I proposition. The three

terms are to have the same relative position as in the first figure. Examples: (1) Find premisses for the conclusion 'no prophet is infallible'; here the term 'man' will do as a middle term; and the required premisses are 'no man is infallible' and 'all prophets are men.' (2) Find premisses for the conclusion 'some elements are metals'; here the term 'undecomposable substances conducting heat and electricity' will do as a middle term; and the premisses required are 'all undecomposable substances conducting heat and electricity are metals,' and 'some elements are undecomposable substances conducting heat and electricity.'

3. *To draw the conclusion, if any, which follows from two given propositions as premisses:*—See if the two premisses are in any particular valid mood in any of the four figures. If so, draw the conclusion which follows from them in accordance with that mood. If not, try to reduce them to a valid mood by verbal changes and by processes of immediate inference. If they can be thus transformed into a valid mood, draw the inference justified by that mood. If they cannot be so transformed, no conclusion follows from the two given propositions. It should be remembered that the conclusion not being given, it is not known which term is major and which minor, that the premiss stated first is not necessarily the major premiss, and the premiss stated second the minor premiss, that the two premisses may be given and taken in any order.

Examples.

(1) All B is A,
No C is not-B.

Here the two premisses are not in any particular valid mood, and seem to involve the fallacy of four terms. But, by permuting the second premiss, we obtain the following syllogism in *Barbara:*—All B is A; all C is B; ∴ all C is A.

(2) No C is not-B,
No B is not-A.

Here the two premisses are negative, and do not seem to justify any conclusion whatever. But, by permuting both, we get the following syllogism in *Barbara:*—All C is B; all B is A; ∴ all C is A, the first being the minor and the second the major premiss.

(3) No A is B,
No not-B is C.

Converting the first premiss, and permuting the converse of the second, we obtain the following valid syllogism in *Celarent*:—'No B is A; all C is B; ∴ no C is A.'

(4) 'No metal is a compound substance,
Gold is not a non-metal.'

By permuting the first and the second premiss, we get the following syllogism in *Barbara*:—"Every metal is an elementary (not-compound) substance; gold is a metal; therefore gold is an elementary substance."

Examples for Solution.

I.—Supply the premiss suppressed in the following:—

(1) Iron is a metal because it conducts heat and electricity.
(2) Gold is a noble metal because it does not rust.
(3) Air is material because it has weight.
(4) Air is a gas because it is not liquid or solid.
(5) This idea is real because it agrees with the external thing.
(6) Material things exist because they are the objects of my perception.
(7) A is the cause of B because it is its invariable antecedent.
(8) A must have a cause because it is a phenomenon.
(9) B must be a mineral because it has no signs of organization.
(10) C must be a plant because it has root and leaves.
(11) D can not be a bird because it has no feather.
(12) E is the effect of D because it invariably follows D.
(13) H can not be an acid because it has neither hydrogen nor oxygen.

II.—Supply premisses from which each of the following propositions can be inferred syllogistically:—

(1) Some elements are not metals.
(2) Gold is a metal.
(3) Gravity is a force.
(4) No metals are compounds.
(5) Only material bodies gravitate.
(6) Water is a compound body.

(7) Matter is indestructible.
(8) Electricity is not a form of matter.
(9) Silver is an element.
(10) All plants are organized.
(11) No crystal is organized.
(12) Some flowers are not odorous.
(13) Some animals have no power of locomotion.

III.—Draw the conclusion, if any, which follows from each of the following pairs of premisses:—

(1)—(*a*)	No not-A is B. No not-B is C.	(*b*)	No B is A. No C is not-B.
(2)—(*a*)	All B is not-A. No C is not-B.	(*b*)	No A is B. No C is not-B.
(3)—(*a*)	No B is A. Some C is not not-B.	(*b*)	No not-A is B. Some C is not not-B.
(4)—(*a*)	Some B is C. No not-A is B.	(*b*)	All A is B. All C is not-B.
(5)—(*a*)	No not-B is C. No B is A.	(*b*)	No not-C is B. No not-B is A.

(6) All metals conduct heat; all metals conduct electricity.
(7) All birds are oviparous; all birds cannot fly.
(8) Every feeling is a mental phenomenon; every feeling is not a sensation.
(9) If the rays of light reach the eye, a sensation is produced; if a sensation is produced, it is accompanied by a perception.
(10) Every sensation is accompanied by a perception; a sensation is sometimes produced internally without any external object.
(11) Every chemical union is accompanied by the evolution of heat; a chemical union is sometimes accompanied by the evolution of light.
(12) If two substances are rubbed together, heat is produced; if two substances are struck against each other, heat is produced.
(13) If this gas is carbonic dioxide, it will produce turbidity in a solution of lime-water; it does produce turbidity in that solution of lime-water.

(14) This substance is an element; an element is either a metal or a non-metal.

(15) A material body is either solid, liquid, or gaseous; this body is not gaseous.

(16) None but animals are sentient beings; all plants are insentient beings.

(17) Only material bodies gravitate; light does not gravitate.

(18) None but elements are metals, oxygen and chlorine are non-metals.

CHAPTER VI.

OF TRAINS OF SYLLOGISTIC REASONING.

§ 1. A Train of Syllogistic Reasoning is a combination of two or more syllogisms so connected with one another as to establish a single conclusion. When each of the component syllogisms is fully expressed, it has either of these two typical forms:

(1) That in which the single conclusion is stated last, and the conclusion in one syllogism forms a premiss in the next.

(2) That in which the single conclusion is stated first, and a premiss in one syllogism forms the conclusion in the next, or both premisses form conclusions in two distinct syllogisms.

First Form.	*Second Form.*
(1) All A is B ... (minor),	(1) All A is E ... (conclusion),
All B is C ... (major),	∵ All D is E ... (major),
∴ All A is C ... (conclusion),	All A is D ... (minor),
(2) All A is C ... (minor),	(2) All A is D ... (conclusion),
All C is D ... (major),	∵ All B is D ... (major),
∴ All A is D ... (conclusion),	All A is B ... (minor),
(3) All A is D ... (minor),	(3) All D is E ... (conclusion),
All D is E ... (major),	∵ All C is E ... (major),
∴ All A is E ... (conclusion).	All D is C ... (minor).

In the example of the first form the single conclusion is "All A is E" stated last, and the conclusion of the first syllogism is a premiss in the second, and the conclusion of the second a premiss in the third.

In the example of the second form, the single conclusion is the same (All A is E), but it is stated first, and the two premisses of the 1st syllogism form the conclusions in the 2nd and 3rd, *i. e.*, are proved by them.

The first syllogism in the first form is called a Prosyllogism in relation to the 2nd, and the 2nd in relation to the 1st is called an Episyllogism; that is, a Prosyllogism is a syllogism in a train of reasoning, whose conclusion forms a premiss in another, and an Episyllogism is a syllogism which has for one of its premisses the conclusion of another. These two terms are relative, and the same syllogism may be a prosyllogism in relation to one, and an episyllogism in relation to another. For example, the 2nd syllogism stands in the twofold relation to the 3rd and the 1st respectively.

In the example of the second form, the 1st syllogism is an episyllogism in relation to the 2nd and 3rd, and both these are prosyllogisms in relation to the 1st.

The first form is called Synthetic, Progressive, or Episyllogistic, because the advance in the reasoning is from a prosyllogism to an episyllogism, from certain premisses to the conclusion which follows from them. The second form is called Analytic, Regressive, or Prosyllogistic, because the advance in the reasoning is from an episyllogism to a prosyllogism, from a conclusion to the premisses which prove it.

§ 2. The synthetical train of syllogistic reasoning gives rise to the Synthetical Method, and the analytical train of syllogistic reasoning to the Analytical Method in Deductive Logic.

In the Synthetical Method we start with certain principles as premisses; and by comparing and combining them in various ways, we deduce the conclusions which follow necessarily from them. In the Analytical Method, on the contrary, we start with the conclusions, and proceed regressively to the principles from which they follow deductively. It is by the former method that Euclid proves his propositions; he starts with the axioms, postulates, and definitions as premisses, and proves *progressively* the propositions which follow from them.

§ 3. An episyllogistic or synthetic train of reasoning in which all the conclusions, except the last, are suppressed, is called a *Sorites*. Thus, omitting the conclusions of the 1st two syllogisms, and consequently also the minor premisses of the 2nd and 3rd in the example given above, we get a Sorites of the following form :—

All A is B,
All B is C,
All C is D,
All D is E,
∴ All A is E,

in which the conclusion of the prosyllogism forms the minor premiss in the next episyllogism. This is called the *Aristotelian Sorites*. When the conclusion of the Prosyllogism forms, on the other hand, the major premiss in the next Episyllogism, we have a sorites of a different form, called, after its discoverer, the *Goclenian Sorites*. In the following train of syllogistic reasoning :—

1.		All B is C	...	(major premiss),
		All A is B	...	(minor premiss),
	∴	All A is C	...	(conclusion),
2.		All A is C	...	(major premiss),
		All D is A	...	(minor premiss),
	∴	All D is C	...	(conclusion),
3.		All D is C	...	(major premiss),
		All E is D	...	(minor premiss),
	∴	All E is C	...	(conclusion).

The conclusion of the 1st syllogism forms the major premiss in the 2nd, and the conclusion of the 2nd the major premiss in the 3rd. Suppressing all the conclusions except the last, and consequently also all the major premisses except the first, we have the following *Goclenian Sorites* :—

All B is C,
All A is B,
All D is A,
All E is D,
∴ All E is C.

Taking the following train of syllogistic reasoning :—

(1)	All D is E	...	(major premiss),
	All C is D	...	(minor premiss),
	∴ All C is E	...	(conclusion),
(2)	All C is E	...	(major premiss),
	All B is C	...	(minor premiss),
	∴ All B is E	...	(conclusion),
(3)	All B is E	...	(major premiss),
	All A is B	...	(minor premiss),
	∴ All A is E	...	(conclusion),

and suppressing all the conclusions except the last, and therefore also all the major premisses except the first, we have the following example of the *Goclenian Sorites :—*

All D is E,
All C is D,
All B is C,
All A is B,
∴ All A is E.

Both the Goclenian and the Aristotelian Sorites are abridged trains of syllogistic reasoning, and both are synthetic, progressive, or episyllogistic, the advance in the reasoning being from a prosyllogism to an episyllogism.

An *Epicheirema* is a prosyllogistic, analytical, or regressive train of reasoning with some of its premisses suppressed. It consists of a syllogism with a reason or reasons for one or both of its premisses being given. For example, the train of reasoning "All A is B; and all C is A, because all C is D: therefore all C is B" is an Epicheirema, in which a reason is given for one premiss, and which may be thus fully expressed :—

(1)	All A is B	...	(major premiss),
	All C is A	...	(minor premiss),
	∴ All C is B	...	(conclusion).

For the minor premiss the reason given is that 'All C is D.'

This with that premiss evidently constitutes an enthymeme, whose major premiss is suppressed, thus :—

(2) All D is A ... (the suppressed major premiss),
All C is D ... (the reason given),
∴ All C is A.

In the following example reasons are given for both the premisses: "All A is B, because all A is G; all C is A, because all F is A; therefore all C is B." When fully expressed it consists of the following three syllogisms :—

(1) All A is B ... (major premiss),
All C is A ... (minor premiss),
∴ All C is B ... (conclusion).

The major premiss is proved by an enthymeme, whose major premiss is suppressed :—

(2) All G is B ... (the suppressed major premiss),
All A is G ... (the reason given),
∴ All A is B ... (conclusion).

The minor premiss is also proved by an enthymeme, whose minor premiss is suppressed :—

(3) All F is A ... (the reason given),
All C is F ... (suppressed minor),
∴ All C is A ... (conclusion).

The *Epicheirema* is thus an abridged train of syllogistic reasoning, in which the argument proceeds analytically, from an episyllogism to a prosyllogism.

The analytic train of syllogistic reasoning which we have given at the beginning of this chapter may give rise to any of the following Epicheiremas by suppressing different premisses :—

(1) All A is D, ∵ all A is B,
All D is E, ∵ all C is E,
∴ All A is E.

(2) All A is D, ∵ all A is B,
All D is E, ∵ all D is C,
∴ All A is E.

(3) All A is D, ∵ all B is D,
All D is E, ∵ all C is E,
∴ All A is E.

(4) All A is D, ∵ all B is D,
All D is E, ∵ all D is C,
∴ All A is E.

In (1) the major premiss of the second syllogism and the minor of the third are suppressed.

In (2) the major premiss of the second and the major premiss of the third syllogism are suppressed.

In (3) the minor premiss of the second syllogism and the minor of the third are suppressed.

In (4) the minor premiss of the second syllogism and the major of the third are suppressed.

The different varieties of trains of syllogistic reasoning are shown in the following tabular view :—

TRAINS OF SYLLOGISTIC REASONING.

- Synthetic, Progressive, or Episyllogistic (from Pro- to Epi- syllogism).
  - Fully-expressed.
  - Abridged (or Sorites).
    - Aristotelian,
      All A is B,
      All B is C,
      All C is D,
      All D is E,
      ∴ All A is E.
    - Goclenian,
      All D is E,
      All C is D,
      All B is C,
      All A is B,
      ∴ All A is E.
- Analytic, Regressive, or Prosyllogistic (from Epi- to Pro- syllogism).
  - Fully-expressed.
  - Abridged (or Epicheirema).
    - Simple, with reason for one premiss only.
    - Double, with reasons for both the premisses.
    - Complex, with reasons for the reasons.

§ 4. Symbolical examples of Sorites with analyses :—

FIRST FIGURE.

Barbara.

Aristotelian.

I. All A is B,
All B is C,
All C is D,
∴ All A is D.

Analysis of I.

(1) All A is B ... (minor),
All B is C ... (major),
∴ All A is C ... (conclusion),

(2) All A is C ... (minor),
All C is D ... (major),
∴ All A is D ... (conclusion).

Goclenian.

I. All C is D,
All B is C,
All A is B,
∴ All A is D.

Analysis of I.

(1) All C is D ... (major),
All B is C ... (minor),
∴ All B is D ... (conclusion),

(2) All B is D ... (major),
All A is B ... (minor),
∴ All A is D ... (conclusion).

Darii.

II. Some A is B,
All B is C,
All C is D,
∴ Some A is D.

II. All C is D,
All B is C,
Some A is B,
∴ Some A is D.

N.B.—Analysis of II. is similar to that of I.

Celarent.

III. All A is B,
All B is C,
No C is D,
∴ No A is D.

Analysis of III.

(1) All A is B ... (minor),
All B is C ... (major),
∴ All A is C ... (conclusion),

(2) All A is C ... (minor),
No C is D ... (major),
∴ No A is D ... (conclusion).

III. No C is D,
All B is C,
All A is B,
∴ No A is D.

Analysis of III.

(1) No C is D ... (major),
All B is C ... (minor),
∴ No B is D ... (conclusion).

(2) No B is D ... (major),
All A is B ... (minor),
∴ No A is D ... (conclusion).

Ferio.

IV. Some A is B,
All B is C,
No C is D,
∴ Some A is not D.

IV. No C is D,
All B is C,
Some A is B,
∴ Some A is not D.

In the 1st figure one premiss only can be particular: the 1st in the Aristotelian and the last in the Goclenian; and only one premiss negative: the last in the former and the first in the latter. It should be observed that, when the conclusion is the same, the order of the premisses in one form is exactly the reverse of that in the other; that is, the conclusion being the same in both, the premisses in the Goclenian are those of the Aristotelian from the bottom upwards. This has given rise to the mistaken notion that the latter is progressive, while the former is regressive; but we have seen that both are equally progressive or episyllogistic. The order of the terms should also be noted. In the Aristotelian the predicate in one premiss becomes the subject in the next, while in the Goclenian the subject in one premiss becomes the predicate in the next.

SECOND FIGURE.

Aristotelian.

V. All A is B,
All B is C,
All C is D,
No E is D,
∴ No A is E.

Analysis of V.

(1) All A is B ... (minor),
All B is C ... (major),
∴ All A is C ... (conclusion),

(2) All A is C ... (minor),
All C is D ... (major),
∴ All A is D ... (conclusion),

(3) All A is D ... (minor),
No E is D ... (major),
∴ No A is E ... (conclusion).

Goclenian.

V. No E is D,
All C is D,
All B is C,
All A is B,
∴ No A is E.

Analysis of V.

(1) No E is D ... (major),
All C is D ... (minor),
∴ No C is E ... (conclusion),

(2) No C is E ... (major),
All B is C ... (minor),
∴ No B is E ... (conclusion),

(3) No B is E ... (major),
All A is B ... (minor),
∴ No A is E ... (conclusion).

In these examples only one syllogism is in the second figure; the others are in the first figure. In the Aristotelian the last,

and in the Goclenian the first, are in the mood *Cesare* of the second figure; all the others are in the first figure.

It should be noted that in the Aristotelian Sorites the conclusion of a Prosyllogism becomes the minor premiss, while in the Goclenian it becomes the major premiss, in the next Episyllogism, throughout the whole train of reasoning. We shall conclude with an Aristotelian Sorites in the 3rd figure:—

VI. All A is B,
All B is C,
All C is D,
All A is E,
∴ Some D is E.

Analysis of VI.

(1) All A is B,
All B is C,
∴ All A is C,

(2) All A is C,
All C is D,
∴ All A is D,

(3) All A is D,
All A is E,
∴ Some D is E.

Here the 3rd Syllogism is in *Darapti* in the 3rd figure, and the others in *Barbara*[1].

§ 5. Questions and exercises.

1. Analyse and test the following trains of reasoning:—

(1) "Bucephalus is a horse; a horse is a quadruped; a quadruped is an animal; an animal is a substance: therefore Bucephalus is a substance."

(2) "If Harpagon be avaricious, he is intent on gain; if intent on gain, he is discontented; if discontented, he is unhappy; now Harpagon is avaricious: he is, therefore, unhappy."

(3) "Whatever promotes happiness is good; whatever perfects the soul promotes happiness: therefore whatever perfects the soul is good; misfortune which happens to the good, serves either to disci-

[1] See Appendix H.

pline or to improve the soul: hence misfortune which befalls the good is good."

(4) "Sentient beings seek happiness; all finite beings are sentient; all men are finite beings; Caius is a man: therefore he seeks happiness."

(5) "That which thinks is active; that which is active has strength; that which has strength is a substance; the soul thinks: therefore it is a substance."

(6) A is equal to B; B is equal to C; C is equal to D; D is equal to E: therefore A is equal to E.

(7) A is greater than B; B is greater than C; C is greater than D; D is greater than E: therefore A is greater than E.

(8) A is the cause of B; B is the cause of C; C is the cause of D; D is the cause of E: therefore A is the cause of E.

(9) A lies above B; B lies above C; C lies above D: therefore A lies above D.

(10) A co-exists with B; B co-exists with C; C co-exists with D: therefore A co-exists with D.

(11) A is a mark of B; B is a mark of C; C is a mark of D: therefore A is a mark of D.

(12) If a gas is heated, its temperature rises; if its temperature rises, its elastic force increases; if its elastic force increases, the pressure on the walls of the containing vessel increases: therefore if a gas is heated, the pressure on the walls of the containing vessel increases.

2. Analyse the demonstration of the 20th Proposition in Todhunter's Euclid, p. 23, into the constituent syllogisms.

3. Prove both synthetically and analytically the 18th Proposition of Euclid, Book I, Todhunter, p. 22.

4. Analyse into fully-expressed syllogisms both the construction and the demonstration of the 32nd Proposition of Euclid, Book I.

5. Distinguish between the Analytical Method in Deductive Logic and Analysis as employed in Geometry.

CHAPTER VII.

Of Fallacies.

§ 1. I.—A General Outline.

A Fallacy, in the proper sense of the term, is a transgression of a rule of inference. A fallacious reasoning is, in fact, an apparent reasoning involving the breach of some rule or other of the various kinds of inference. Thus there are as many different kinds of Fallacies as of Reasoning or Inference.

The breach of the laws of Inductive Reasoning gives rise to the Fallacies of Induction with which we have nothing to do here. The breach of the rules of Deductive Inference gives rise, first, to the Fallacies of Immediate Inference, when the rule transgressed is a rule of Immediate Inference, and, secondly, to the Fallacies of Mediate Inference, when the rule violated is one of Syllogism or of any other kind of Mediate Deductive Reasoning. Thus we have the following classes of Fallacies in the sense we have defined above:—

INFERENTIAL FALLACIES.

- Inductive (arising from the transgression of the canons of inductive reasoning).
- Deductive (arising from the transgression of the rules of deductive reasoning).
    - Fallacies of Immediate Inference.
        - Fallacies of Conversion.
        - " Permutation.
        - " Contraposition.
        - " Opposition.
        - " Subalternation.
        - " Modal Consequence.
        - " Change of Relation.
    - Fallacies of Mediate Inference.
        - I. Syllogistic:—
            - Undistributed Middle.
            - Illicit Process.
            - Four Terms.
            - &c. &c.
        - II. Non-Syllogistic, *e. g.*, Mathematical.

In a wider sense a Fallacy is a transgression of any logical rule whatever. In this sense we have in Deductive Logic the Fallacies or Faults of Division and Definition; and in Inductive Logic those of Classification, Hypothesis, &c. The violation of the rules to which every logical division and definition ought to conform gives rise to the faults of division and definition, such as cross division, incomplete division, definition by accidental qualities, &c. To this class belong also the fallacies arising from ambiguity in language, such as those of Ambiguous Middle, of Division, Composition, &c. These are transgressions of the logical rule that our thoughts should be expressed and reasonings conducted in clear and unambiguous language.

NON-INFERENTIAL LOGICAL FALLACIES.

- Those usually treated in Deductive Logic.
  - Those arising from the transgression of the rules of Definition and Division.
    - Faults of Definition:—
      - (1) Description or definition by *accidental* qualities; redundant definition.
      - (2) Too narrow or too wide definition.
      - (3) The *circle* in definition, or definition by synonyms.
      - (4) Obscure, figurative, and ambiguous definitions.
      - (5) Negative definition.
    - Faults of Division:—
      - (1) Physical Partition and Metaphysical Analysis.
      - (2) Cross Division.
      - (3) Incomplete or Overcomplete (too narrow or too wide) Division.
      - (4) Overlapping Division.
  - Those arising from ambiguous language, called *Semi-logical*:—
    - Ambiguous Middle.
    - Fallacy of Composition.
    - " Division.
    - " Accident.
    - &c. &c.
- Those usually treated in Inductive Logic, arising from the transgression of the rules of Classification, Hypothesis, Nomenclature, &c.

In the widest sense, the word *fallacy* may be taken to mean an error of any kind, whether of Intuition, Perception, Observation, Division, Definition, Inference, &c. In this sense it includes, besides those mentioned above, the fallacies of Irrelevancy or Irrelevant Conclusion, technically called *Ignoratio Elenchi*, of *Petitio Principii* (begging the question), of False Premiss, and also those which Mill calls Fallacies of Simple Inspection, or of Erroneous First Principles and Axioms.

NON-LOGICAL OR MATERIAL FALLACIES.

- Premiss unduly assumed.
  - Premisses depending on the conclusion, *Petitio Principii.*
    - Premiss the same as the conclusion, the argument in a circle.
    - Premiss unfairly implying the conclusion.
  - Premiss false or unsupported; *Non causa pro causa;* Erroneous First Principles and Axioms, &c. &c.
- Irrelevant conclusion or *Ignoratio Elenchi* (the argument or conclusion not to the point).
  - Fallacies of appealing to the passions: *Argumentum ad hominem*, ,, *populum*, ,, *verecundiam.*
  - Fallacy of shifting ground, &c.

§ 2. II.—Fallacies in Deductive Logic.

It is not necessary that we should describe and explain in detail each of the fallacies mentioned above, for most of them have been already made evident in explaining and illustrating the rules. In the following pages we shall notice and illustrate the more frequent and important kinds only.

A.—Logical Fallacies.

1. *Inferential.*

(1)—Fallacies of Immediate Inference.

In Conversion the most frequent fallacy is the simple conversion of A: 'All A is B, ∴ All B is A,' 'If A is, B is, ∴ If B is, A is.' The inference is, of course, fallacious, and violates the rule of conversion, viz., that no term should be distributed in the converse which was not distributed in the convertend; and the valid inference is 'Some B is A,' 'In some cases if B is, A is.' The simple conversion of O is also fallacious for the same reason: 'Some A is not B, ∴ Some B is not A.' The conversion of O into 'Some not-B is A' is not admissible, because it violates the first rule of conversion, viz., that the subject and the predicate of the convertend should be, respectively, the predicate and the subject in the converse.

In Obversion, Æquepolence, or Permutation the following are fallacious:—

(1) All A is B; ∴ All not-A is not-B.
(2) All metals are elements;
∴ All not-metals are not-elements.
(3) Cold is agreeable;
∴ Heat is disagreeable.
(4) Virtue will be rewarded;
∴ Vice will be punished.

In Contraposition the following are fallacious:—

(1) No A is B; ∴ All not-B is A.
(2) No man is perfect;
∴ All imperfect beings are men.
(3) Some A is B; ∴ Some not-B is A.
(4) Some elements are metals;
∴ Some not-metals are elements.

In Opposition the following are fallacious:—

(1) 'All plants are flowerless' is false;
∴ 'No plants are flowerless' is true.

(2) 'All philosophers are poets' is false;
∴ 'No philosophers are poets' is true.
(3) 'Some plants can move' is true;
∴ 'Some plants cannot move' is true.
(4) 'Some elements are metals' is true;
∴ 'Some elements are not metals' is true.
(5) 'Some men are wise' is true;
∴ 'Some men are not wise' is false.

§ 3. (2)—Fallacies of Syllogistic Inference.

These arise from the transgression of the syllogistic rules. Everyone of them is *ultimately* a breach of some one or other of the fundamental principles of Deductive Logic, and *proximately* of the general syllogistic rules, or of the special rules for each figure. Regarded as transgressions of the nine general syllogistic rules we have given in Part III. Chap. III. the fallacies are as follows:—

(1) The Fallacy of *Four Terms*, arising from the transgression of the 1st rule.

(2) The Fallacy of *Four Premisses*, arising from the violation of the 2nd rule.

(3) The Fallacy of *Undistributed Middle*, arising from the breach of the 3rd rule.

(4) The Fallacy of *Illicit Process*, arising from the transgression of the 4th rule: of the *Major Term*, when this term is distributed in the conclusion and not in the premiss; and of the *Minor Term*, when this term is distributed in the conclusion and not in the premiss.

(5) The Fallacy of *Negative Premisses*, arising from the violation of the 5th rule.

(6) The Fallacies of *Affirmative Conclusion from a Negative Premiss*, and *Negative Conclusion from Affirmative Premisses*, arising from the violation of the 6th and 7th rules.

(7) Fallacies also arise from the transgression of the 8th and 9th rules, and belong to one or other of the fallacies mentioned above.

The most important of the fallacies under this head are those

of Undistributed Middle and Illicit Process. Of these we shall give the following examples:—

1. The virtuous are happy,
   The wealthy are happy;
   ∴ The wealthy are virtuous.

Undistributed Middle, because the middle term being the predicate in the two affirmative premisses, is not distributed.

2. All material bodies are extended,
   Shadows are extended;
   ∴ Shadows are material bodies.

Undistributed Middle.

3. Whatever thinks exists,
   Matter does not think;
   ∴ Matter does not exist.

Illicit Process of the Major Term,

which being the predicate in the affirmative major premiss, is undistributed, but which is distributed in the conclusion, being the predicate in a negative proposition.

4. All material bodies have weight,
   All material bodies are extended;
   ∴ All extended things have weight.

Illicit Process of the Minor Term,

which is distributed in the conclusion, but not distributed in the minor premiss.

5. All men are mortal,
   All men are rational;
   ∴ All rational beings are mortal.

Illicit Process of the Minor Term.

6. All metals conduct heat and electricity,
   All metals are elements;
   ∴ All elements conduct heat and electricity.

Illicit Process of the Minor Term.

7. All Hindus are Aryans,
   The Persians are not Hindus;
   ∴ The Persians are not Aryans.

Illicit Process of the Major Term.

2. *Non-Inferential.*

§ 4. (1)—Semi-logical Fallacies.

These arise from ambiguous language. If a term is ambiguous, it is really equivalent to two, and there is thus the fallacy of *four terms.* In a fallacy of this kind, it is the middle term that is generally ambiguous, giving rise to what is called the fallacy of *ambiguous middle.* In some cases, the middle term is taken distributively in the major premiss, and collectively in the minor; in some it is taken collectively in the major and distributively in the minor premiss. In the former, we have the Fallacy of *Composition,* and in the latter the Fallacy of *Division.* We shall now give a few examples of each of these varieties:—

1. An organized body is either a plant or an animal; a nation is an organized body: therefore a nation is either a plant or an animal. Here the word *body* is ambiguous.

2. Light is a mode of motion; feather is light: therefore feather is a mode of motion. Here the double meaning of the word *light* is obvious.

3. "All cold is to be expelled by heat; this person's disorder is a cold: therefore it is to be expelled by heat." Here the word *cold* is ambiguous: in the first premiss it means a low degree of heat or the sensation of coldness, and in the second a particular bodily disorder.

4. "Projectors are unfit to be trusted; this man has formed a project: therefore this man is unfit to be trusted." Here *projector* and *formed a project* do not mean the same thing.

5. "To be acquainted with the guilty is a presumption of guilt; this man is so acquainted: therefore we may presume that he is guilty." Here the phrases '*presumption of guilt*' and '*presume that he is guilty*' have different significations.

6. "All the angles of a triangle are equal to two right angles, ABC is an angle of a triangle; ∴ ABC is equal to two right angles," is a Fallacy of *Division;* for the middle term is taken *collectively* in the major and *distributively* in the minor premiss.

7. "Five is one number; three and two are five: therefore three and two are one number," is also a Fallacy of *Division*.

8. "Three and two are two numbers; five is three and two: therefore five is two numbers," is a Fallacy of *Composition;* for the middle term is taken *distributively* in the major premiss, and *collectively* in the minor.

9. "All the angles of a triangle are less than two right angles, ABC, ACB, and BAC are all the angles of a triangle; ∴ they are less than two right angles."

Here the word *all* is ambiguous. In the major premiss the term '*all the angles of a triangle*' is taken *distributively* to mean any angle. In the minor premiss, it is doubtful whether it is taken collectively or distributively. If it is taken collectively, the argument involves the Fallacy of *Composition*. If it is taken distributively, the argument is valid.

10. "I can afford to buy these books. I can afford to buy these pictures. I can afford to buy these statuettes. The books, the pictures, and the statuettes are all that I, at present, wish to purchase. I can, therefore, buy everything that I want to buy." This is a Fallacy of *Composition;* 'these books,' 'these pictures,' and 'these statuettes' are taken *distributively* or separately in the first premiss, and *collectively* or jointly in the second.

11. The Fellows of the Royal Society have made the greatest discoveries in Science; A, B, and C are Fellows of the Royal Society; therefore A, B, and C have made the greatest discoveries in Science. This is a Fallacy of *Division*.

The next fallacy under this head is the Fallacy of *Accident*, which consists in taking a term *simply* or without any condition in one premiss, and as *modified by certain accidents* or as under certain circumstances in the other. For example, "What is bought in the market is eaten, raw meat is bought in the market; therefore raw meat is eaten." In the minor premiss the middle term, *bought in the market*, is taken *simply*, while in the major premiss it must be understood as *modified by certain accidents* or qualities not present in the other. There are, in fact, two middle terms, one 'bought in the market' without anything understood

after it, and the other 'bought in the market' with some such phrase as 'and cooked at home' understood after it. The Fallacy of Accident as defined above includes both the *Fallacia a dicto simpliciter ad dictum secundum quid* and the *Fallacia a dicto secundum quid ad dictum simpliciter*,—that is, both the fallacy of arguing from a simple statement to a statement under a certain condition, and the fallacy of arguing from a statement under a certain condition to a simple statement. There is another fallacy of a similar nature, in which the reasoning proceeds from a statement under a certain condition to a statement under a different condition. All the three forms of the Fallacy of Accident are due to the *ambiguity of language*, and may be easily avoided by stating the meaning of the propositions in clear and unambiguous language.

In conclusion, it appears that all the different kinds of Semi-logical Fallacies arising from ambiguous language are really different forms of the fallacy of Ambiguous Middle, and, in rare cases, of Ambiguous Extreme. In some the middle term is in itself ambiguous, having two meanings in the same form or in different forms or parts of speech. In others the ambiguity arises from some of the words being ambiguous, or from the grammatical structure of the sentence being ambiguous. But an ambiguous term, whatever be the source of its ambiguity, is really equivalent to two terms; and all the forms of ambiguous middle and of ambiguous extreme are really transgressions of the first syllogistic rule, that is, they are all *fallacies of four terms*.

§ 5. (2) The fallacies or faults arising from the violation of the rules of Logical Division and Definition have been already explained under those subjects, and do not require any separate treatment here.

B.—NON-LOGICAL OR MATERIAL FALLACIES.

These fallacies do not properly belong to Deductive Logic, as they are concerned about the subject-matter of reasoning. The more important of them are: (1) the *Petitio Principii*, including

the 'Argument in a Circle,' and 'Begging the Question'; (2) the Falsity of Premiss; and (3) the *Ignoratio Elenchi*, or the Fallacy of Irrelevancy, or, as it is sometimes called, the Irrelevant Conclusion.

§ 6. (I)—Of the *Petitio Principii.*

This fallacy in its simplest form occurs when a proposition is proved by another proposition, and this other is again proved by the first. For example, 'A is, because B is; and B is, because A is.' Here the conclusion is proved by the premiss, and the premiss by the conclusion; and the fallacy is quite evident, and consists really in proving 'A is' by 'A is,'—the same by the same, *idem per idem.*

In the following example, the major premiss of the 1st syllogism is proved by the 2nd, and the major premiss of the 2nd by the 1st syllogism:—

I. 1. M is P,	2. S is P,
S is M;	M is S;
∴ S is P.	∴ M is P.

Here 'S is P' is proved by a syllogism whose major premiss is 'M is P,' and this premiss is proved by a syllogism whose major premiss is 'S is P.' Thus, 'S is P' is proved with the aid of 'M is P,' and 'M is P' is proved with the aid of 'S is P': therefore 'S is P' is proved by 'S is P.' In this also the fallacy is almost quite evident. But if the two syllogisms here placed one after the other were, respectively, the first and the last of a long train of reasoning, it would not be so easy to detect the fallacy. And this difficulty is still further increased partly by the difference of language in which the same proposition may occur in different parts of the train, and partly by the omission of many intervening syllogisms. For example—

II. 1. A is B,	3. A is D,
B is C;	D is E;
∴ A is C.	∴ A is E.
2. A is C,	4. A is E,
C is D;	E is B;
∴ A is D.	∴ A is B.

In this train of reasoning the final conclusion in the 4th syllogism is the same as the minor premiss of the 1st, that is, this premiss is proved by the 4th syllogism. But how is this final conclusion established? By using as a premiss the proposition 'A is E,' which has been itself proved by taking the final conclusion 'A is B' as a premiss in the first syllogism. Thus the final conclusion is really established by taking itself as a premiss in a part of the train of reasoning.

In the 1st syllogism, 'A is C' is proved by taking 'A is B' as a premiss.

In the 2nd, 'A is D' is proved by taking 'A is C' as a premiss, and therefore by indirectly taking 'A is B' as a premiss.

In the 3rd, 'A is E' is proved by taking 'A is D' as a premiss, and therefore by taking indirectly 'A is B' as an ultimate premiss.

In the 4th, 'A is B' is proved by taking 'A is E' as a premiss, and therefore by taking indirectly 'A is B' as an ultimate premiss. That is, 'A is B' is proved by 'A is B.'

Or the fallacy may be exposed thus:—A is C, because A is B; and A is B, because A is E (4th syllogism), and A is E, because A is D (3rd syllogism), and A is D, because A is C (2nd syllogism), therefore A is B, because A is C. Thus 'A is B' is proved by 'A is C,' and 'A is C' is proved by 'A is B.' Here the use of the symbols has enabled us to detect the fallacy easily; but if the language of the last syllogism were different from that of the first, and if, moreover, some of the intervening syllogisms were suppressed, the train being much longer than that represented above, it would not be so easy to detect the fallacy, and expose it by analysing the whole train.

The *Petitio Principii* in the *stricter* sense may, then, be defined as a fallacy in which the conclusion is proved by means of itself, or in which the conclusion is the same as one of the premisses. In the *wider* sense it includes also those fallacies in which the conclusion follows from, or is presupposed by, one premiss *independently* of the others. For example—

III. All men are mortal,
Those who are mortal are not immortal;
∴ No man is immortal.

In order to prove the conclusion 'No man is immortal,' two premisses are advanced, and the argument is apparently stated in the form of a syllogism; but the conclusion really follows immediately from, or is presupposed by, the first or minor premiss 'All men are mortal,' which *obverted* gives the conclusion directly.

In the stricter sense, the *Petitio Principii* is called the *Argument in a Circle* because the final conclusion is the same as the first premiss, because the reasoning coming back whence it started, completes a circle. In the wider sense, including all forms, it is called *Begging the Question*, because it begs or surreptitiously takes for granted a proposition which is identical in meaning with, or is a consequence of, the very proposition to be proved.

§ 7. (2) Of the Falsity of Premiss.

The next fallacy under this head is the Falsity of Premiss. This fallacy occurs when one of the premisses is false; when something is regarded as a cause of an event, which is really not the cause, which is either merely a sign or an antecedent of it. It is also called *Non causa pro causa*, the assuming as a cause that which is not a cause, and *Post hoc ergo propter hoc*, or after this, and therefore on account of, or caused by, this.

Whately thus distinguishes the *Petitio Principii* from *Non causa pro causa:* "Let the name then of 'petitio principii' (*begging the question*)," he says, "be confined to those cases in which one of the Premisses either is manifestly the same in sense with the Conclusion, or is actually proved from it, or is such as the persons you are addressing are not likely to know, or to admit, except as an inference from the Conclusion; as, for example, if any one should infer the authenticity of a certain history, from its recording such and such facts, the reality of which rests on the evidence of that history. All other cases in which a Premiss (whether the expressed or the suppressed

ponents' proposition, as for the apparent establishment of your own; for it is substantially the same thing to *prove* what was not denied, or to *disprove* what was not asserted. The latter practice is not less common; and it is more offensive, because it frequently amounts to a personal affront, in attributing to a person opinions, &c., which he perhaps holds in abhorrence. Thus, when in a discussion one party vindicates, on the ground of expediency, a particular instance of resistance to Government in a case of intolerable oppression, the opponent may gravely maintain, that 'we ought not to do evil that good may come'—a proposition which of course had never been denied; the point in dispute being 'whether resistance in this particular case *were* doing evil or not.' Or again, by way of disproving the assertion of the '*right* of private judgment in religion,' one may hear a grave argument to prove that 'it is impossible that every one could be *right in his judgment*.' In these examples, it is to be remarked that the fallacy of *Petitio Principii* is combined with that of *Ignoratio Elenchi;* which is a very common and often successful practice,—viz., the Sophist proves, or disproves, not the proposition which is really in question, but one which is so dependent on it as to proceed on the supposition that it is already decided, and can admit of no doubt; by this means his 'assumption of the point in question' is so indirect and oblique, that it may easily escape notice; and he thus establishes, practically, his conclusion, at the very moment he is withdrawing your attention from it to another question. For example, an advocate will prove, and dwell on the high *criminality* of a certain act, and the propriety of severely punishing it; assuming (instead of proving) the *commission*.

"There are certain kinds of arguments recounted and named by logical writers which we should by no means universally call Fallacies; but which *when unfairly* used, and *so far as they are* fallacious, may very well be referred to the present head; such as the *Argumentum ad hominem* (or personal argument), *Argumentum ad verecundiam*, *Argumentum ad populum*, &c., all of them regarded as contradistinguished from *Argumentum ad rem*

or *ad judicium*. These have all been described in the lax and popular language before alluded to, but not scientifically: the *Argumentum ad hominem*, they say, 'is addressed to the peculiar circumstances, character, avowed opinions, or past conduct of the individual, and therefore has a reference to him only, and does not bear directly and absolutely on the real question, as the *Argumentum ad rem* does'; in like manner, the *Argumentum ad verecundiam* is described as an appeal to our reverence for some respected authority, some memorable institution, &c., and the *Argumentum ad populum* as an appeal to the prejudices, passions, &c., of the multitude; and so of the rest[1]."

"The fallacy of Irrelevant Conclusion (*Ignoratio Elenchi*) is nowhere more common than in protracted controversy, when one of the parties having attempted in vain to maintain his position, ***shifts his ground*** as covertly as possible to another, instead of honestly giving up the point. An instance occurs in an attack made in the system pursued at one of our universities. The objectors finding themselves unable to maintain their charge of the present neglect (viz., in the year 1810) of Mathematics in that place (to which neglect they attributed the late *general decline* in those studies), shifted their ground, and contended that that University 'was never famous for mathematicians'; which not only does not establish, but absolutely overthrows, their own original assertion; for if it *never* succeeded in these pursuits, it would not have caused their late *decline*[2]."

§ 9. Besides the fallacies we have mentioned above, two more, namely, the *Non sequitur* and the *Fallacy of many questions*, are also given under the class of material fallacies. The first occurs when the conclusion does not in any way follow from the premisses, when, in fact, there is no logical connection between the two, anything being inferred from anything else. The second occurs when, by way of asking questions, certain assumptions are made in regard to certain things or persons: "In what subjects did you fail?" This question assumes

[1] Whately's *Elements*, pp. 141—142. [2] *Ibid.* pp. 143—4.

(1) that you appeared at an examination, and (2) that you failed in more than one subject; while the real fact might be just the reverse.

All these fallacies, as I have already said, do not properly belong to Deductive Logic. It is no part of Deductive Logic to ascertain whether a certain premiss is true or false, or whether a conclusion or an argument advanced by a party is irrelevant to the subject in debate. The *Petitio Principii*, indeed, would seem to be a fallacy of Deduction, inasmuch as the rules of Deductive Inference imply that a proposition can not be proved by means of itself, that a proposition, when inferred, must be inferred from others which are severally different from itself

§ 10. Exercises.

1. In testing an argument consisting of a single categorical syllogism, the following method should be followed:—

(i) Find the conclusion and note its subject and predicate which are, respectively, the minor and the major term of the syllogism.

(ii) Find the term which is not in the conclusion. It must be the middle term. (*a*) See if there be any other; if there is, then the argument involves the fallacy of *four terms*. (*b*) See if the middle term be ambiguous; if it is, then there is the fallacy of *ambiguous middle*. (*c*) See whether the middle term be distributed; if it is not, then there is the fallacy of *undistributed middle*.

(iii) Find the premiss which contains the minor term, and the premiss which contains the major term; and these two premisses are, respectively, the minor and the major premiss.

(iv) See if there be any term which is undistributed in either premiss, but distributed in the conclusion. If there is, then there is an *illicit process*.

If there be none of the above fallacies, then the argument is valid. To confirm this,

(v) Find the figure and mood of the syllogism, and see if the mood is a valid one in that figure.

2. In many cases the invalidity of an argument may be detected on mere inspection. For instance, when it contains two particular or two negative premisses, or when the middle term is not distributed, or when one of the premisses is negative and the conclusion affirmative, or, lastly, when one of the premisses is particular and the conclusion universal.

3. The method described above seems, on the whole, to be the best. But there are of course other methods, which may also be applied to verify the result obtained by it or to test the argument independently. For example, the figure and the mood of the syllogism may be at once found; if the mood be a valid one in the particular figure, the syllogism will be valid. Or the figure being found, the syllogism may be tested by the canon or the special rules of that figure; if it conform to the canon or to the rules, it will be valid. Or the syllogism may be tested by the method of the comparison of the diagrams: if the conclusion follow in every case, it will be valid; if it do not follow in a single case, it will be invalid[1].

4. If an argument consists of more than one syllogism, that is, of a train of reasoning, it should be analysed into the constituent syllogisms; and each of them should be tested as described above. If any of the premisses be understood or suppressed, they should be supplied, and the constituent syllogisms fully expressed. In the case of Enthymemes, the suppressed premiss, whether true or false, should be supplied. In the case of Dilemmatic and other *mixed* arguments, they should be tested by their rules, and reduced to the categorical form. In the case of Extra-logical or Material fallacies, the student should be able to refer them to their respective classes and show where the fallacy lies.

Examples.

Test the following arguments:—

1. Every metal conducts heat; every metal conducts electricity: therefore every substance that conducts heat conducts electricity.

2. No minerals are plants; no plants are animals: therefore no minerals are animals.

3. All plants are organized; no crystals are plants: therefore no crystals are organized.

[1] Read also the directions given in Part III. Chap. v.

4. All birds are feathered; bats are not birds: therefore bats are not feathered.

5. All feathered animals are birds; bats are not birds: therefore bats are not feathered animals.

6. Only animals are sentient beings; fishes are animals: therefore fishes are sentient beings.

7. None but the Hindoos worship *Shiva;* all Bengalees are Hindoos: therefore all Bengalees worship *Shiva.*

8. All metals except one are solid; this substance is a metal: therefore it is solid.

9. Every object of thought is either an idea of sensation or an idea of reflection; matter is neither: therefore matter is not an object of thought.

10. Every element is either a metal or a non-metal; hydrogen is an element: therefore it is either a metal or a non-metal.

11. Fishes live in water; whales live in water: therefore whales are fishes.

12. Water is liquid; ice is water: therefore ice is liquid.

13. Plato is a philosopher; Plato approves of communism: therefore a philosopher approves of communism.

14. Aristotle believes in the immortality of the rational soul; Aristotle is the greatest intellect ever born: therefore the greatest intellect ever born believes in the immortality of the rational soul.

15. All poets are not imaginative, some philosophers are poets: therefore some philosophers are not imaginative.

16. "The Cretans are liars; A, B, C are Cretans: therefore A, B, C are liars."—Hamilton, Vol. III.

17. Every planet moves round the sun; the earth moves round the sun: therefore the earth is a planet.

18. Knowledge is power; perception is knowledge: therefore perception is power.

19. Cognition is a mental act; cognition is knowledge; knowledge is power: therefore power is a mental act.

20. "Whatever is dictated by nature is allowable; devotedness to the pursuit of pleasure in youth, and to that of gain in old age, are dictated by nature: therefore they are allowable."—Whately.

21. "That man is independent of the caprices of fortune who places his chief happiness in moral and intellectual excellence; a true

philosopher is independent of the caprices of fortune: therefore a true philosopher is one who places his chief happiness in moral and intellectual excellence."—Whately.

22. Give thanks unto the Lord; for he is good; for his mercy endureth for ever.

23. "Some objects of great beauty answer no other perceptible purpose but to gratify the sight; many flowers have great beauty; and many of them accordingly answer no other purpose but to gratify the sight."

24. "War is productive of evil; therefore peace is likely to be productive of good."—Whately.

25. "All that glitters is not gold; tinsel glitters: therefore it is not gold."—Whately.

26. If the rays of light reach the eye, or if the vibrations of sound reach the ear, a sensation is produced; but a sensation is not produced: therefore neither have the rays of light reached the eye, nor have the vibrations of sound reached the ear.

27. Electricity is neither a form of matter nor a form of energy; all material objects are either forms of matter or forms of energy: therefore electricity is not a material object.

28. If two oppositely electrified bodies be brought near, they attract each other; these two bodies repel: therefore they are not oppositely electrified.

29. If two similarly electrified bodies be brought near, they repel each other; these two bodies are not similarly electrified: therefore they do not repel each other.

30. The theory of evolution must be true because every scientific man worthy of the name believes in it.

31. A material body is either solid or fluid; this body is solid: therefore it is not fluid.

32. Every element is either solid or fluid; every element is not fluid: therefore every element is solid.

33. If a chemical union takes place, either heat or light is evolved; if oxygen and nitrogen are united in the proportions in which they exist in the atmospheric air, neither heat nor light is produced: therefore if oxygen and nitrogen are united in those proportions, no chemical union takes place.

34. If Darwin's theory of the origin of species be not true, every

species must be recognized as a special creation; but it is impossible that God should have created so many different species, when he could have easily evolved them all from a few: therefore Darwin's theory of the origin of species is true.

35. Plato is the father of Idealism; Plato is the founder of Political Philosophy: therefore the father of Idealism is the founder of Political Philosophy.

36. "The volume of a body diminishes when it is cooled, because the molecules then become closer."—Ganot's *Popular Physics*.

37. "Impenetrability and extension might be more aptly termed essential attributes of matter, since they suffice to define it."—Ganot.

38. "The struggle for existence reaches even to these little creatures, for they devour still smaller ones."—Ganot.

39. "Since the volume of every body may be diminished, we conclude that all bodies possess physical pores."—Ganot.

40. "No absolute rest is known in the universe; for the earth and the other planets rotate about the sun and about their own axis; and therefore all the parts composing them share this double motion."—Ganot.

41. "Whenever a body is heated, its volume increases, because its molecules are driven apart."—Ganot.

42. Matter is extended because it is impenetrable; and it is impenetrable because every part of it occupies a certain portion of space.

43. "A negro is a man: therefore he who murders a negro murders a man."—Whately.

44. "Meat and drink are necessaries of life; the revenues of Vitellius were spent on meat and drink: therefore the revenues of Vitellius were spent on the necessaries of life."—Whately.

45. "He who calls you a man speaks truly; he who calls you a fool, calls you a man: therefore he who calls you a fool speaks truly.'

46. "Warm countries alone produce wines; Spain is a warm country: therefore Spain produces wines."—Whately.

47. "What we eat grew in the fields; loaves of bread are what we eat: therefore loaves of bread grew in the fields."—Whately.

48. Matter is impenetrable because it is extended; and it is extended because every atom of it, however small in dimensions, must occupy some little space.

49. "We are conscious of one mental state only as we contradistinguish it from another."—Hamilton's *Metaphysics*, Vol. I.

50. "We are conscious of an external world only as we are conscious of it as distinct from others."—Hamilton, Vol. I.

51. Truly we serve, because freely we love.

52. "A judgment is a simple act of mind, for every act of mind implies judgment."—Hamilton, Vol. I. Petitio principii

53. "Every mental phenomenon is either an act of knowledge, or only possible through an act of knowledge, for consciousness is a knowledge—a phenomenon of cognition."—Hamilton, Vol. I.

54. "Certain thoughts are universal, inasmuch as they arise under the same conditions in all men; they are necessary, because their genesis under these conditions is invariable."—Huxley's *Hume*, p. 86.

55. "For those who are bent on cultivating their minds by diligent study, the incitement of Academical honours is unnecessary; and it is ineffectual for the idle, and such as are indifferent to mental improvement: therefore the incitement of Academical honours is either unnecessary or ineffectual."—Whately.

56. "Those who hold that the insane should not be punished ought in consistency to admit also that they should not be threatened; for it is clearly unjust to punish any one without previously threatening him."

57. "If he pleads that he did not steal the goods, why, I ask, did he hide them, as no thief ever fails to do?"

58. "'No one can maintain that all Republics secure good government who bears in mind that good government is inconsistent with a licentious press.' What premisses must be supplied in order to express the above reasoning in Ferio, Festino, and Ferison, respectively?"

59. "If all were capable of perfection, some would have attained it; but, none having done so, none are capable of it."

60. "As thought is existence, what contains no element of thought must be the non-existent."

61. "Since the laws allow everything that is innocent, and avarice is allowed, it is innocent."

62. "Timon being miserable is an evil-doer, as happiness springs from well-doing."

63. "You can not stand still either intellectually or morally; and, therefore, if you are not advancing in the one or the other or both respects, you must be falling back."

64. Nothing and pure being are identical, inasmuch as both are devoid of all qualities.

65. "Theft is a crime; theft was encouraged by the laws of Sparta: therefore the laws of Sparta encouraged crime."—Whately.

66. "Revenge, Robbery, Adultery, Infanticide, &c., have been countenanced by public opinion in several countries; all the crimes we know of are Revenge, Robbery, Adultery, Infanticide, &c.: therefore all the crimes we know of have been countenanced by public opinion in several countries."—Whately.

67. "Every hen comes from an egg; every egg from a hen: therefore every egg comes from an egg."—Whately.

68. "Switzerland is a Republic, and, you will grant, a more stable Power is not to be found; nor, again, is any political society more settled than the United States. Surely, then, Republican France can be in no danger of revolution."

69. "If a conclusion is more certain to be wrong where the reasoning is correct from premisses that are false, will not the best logician be the least safeguard in subjects where perfect certainty is unattainable?"

70. "No one should be punished if he is innocent; this man should not be punished: therefore he is innocent."

71. "Every rule has exceptions; this is a rule, and therefore has exceptions: therefore there are some rules that have no exceptions."

72. "If I am to pass this examination I shall pass whether I do my papers or not; and if I am not to pass, I shall not pass whether I do my papers or not: therefore it is no matter whether or not I do my papers."

73. "A necessary being cannot be the effect of any cause; for if it were, its existence would depend upon that of its cause and would be no longer necessary."

74. "Whatever is conditioned must depend on some cause external to itself; this world is conditioned by time and space: therefore this world depends upon some cause external to itself."

75. "Position we must evidently acknowledge to be relative, for we cannot describe the position of a body in any terms which do not express relation."—Maxwell's *Matter and Motion*, p. 84.

76. If the theory of evolution be true, man is descended from

the lower animals; if the theory of evolution be true, man is not a special creation: therefore if man is not a special creation, he is descended from the lower animals.

77. "The learned are pedants; A is a learned man: therefore A is a pedant."

78. "If it be fated that you recover from your present disease, whether you call in a doctor or not, you will recover; again, if it be fated that you do not recover from your present disease, whether you call in a doctor or not, you will not recover; but one or other of the contradictories is fated: therefore to call in a doctor is of no consequence."—*Vide* Hamilton, Vol. III. pp. 462, 464.

79. "Perception is a cognition or act of knowledge; a cognition is an immanent act of mind; but to suppose the cognition of any thing external to the mind would be to suppose an act of the mind going out of itself, in other words, a transcunt act; but action supposes existence, and nothing can act where it is not: therefore to act out of self is to exist out of self, which is absurd."—Hamilton's *Lectures*, Vol. II. p. 118.

80. "Mind and matter, it is said, are substances, not only of different, but of the most opposite natures; separated, as some philosophers express it, by the whole diameter of being; but what immediately knows must be of a nature correspondent, analogous to that which is known; mind cannot, therefore, be conscious or immediately cognizant of what is so disproportioned to its essence as matter."—Hamilton's *Lectures*, Vol. II. p. 120.

81. "The mind can only know immediately that to which it is immediately present; but as external objects can neither themselves come into the mind, nor the mind go out to them, such presence is impossible: therefore external objects can only be immediately known through some representative object."—Hamilton's *Lectures*, Vol. II. p. 122.

82. "The table, which we see, seems to diminish, as we remove farther from it; but the real table which exists independently of us suffers no alteration: it was, therefore, nothing but its image which was present to the mind."—Hume.

83. "Take, for example, the term *man*. Here we can call up no notion, no idea, corresponding to the universality of the class or term. This is manifestly impossible. For as *man* involves contradictory

attributes, and as contradictory attributes can not co-exist in one representation, an idea or notion adequate to *man* can not be realized in thought."—Hamilton, Vol. II. p. 297.

84. "The class *man* includes individuals, male and female, white and black, copper-coloured, tall and short, fat and thin, straight and crooked, whole and mutilated, &c., &c.; and the notion of the class must, therefore, represent all and none of these. It is, therefore, evident that we can not accomplish this; and this being impossible, we can not represent to ourselves the class *man* by any equivalent notion or idea."—Hamilton, Vol. II. p. 297.

85. "It is manifest, indeed, that man, so far as he is a man for the glory of God, must be an end unto himself, for it is only in the accomplishment of his own perfection that, as a creature, he can manifest the glory of his Creator."—Hamilton, Vol. I. p. 5.

86. "Consciousness supposes a contrast—a discrimination; for we can be conscious only inasmuch as we are conscious of something; and we are conscious of something only inasmuch as we are conscious of what that something is—that is, distinguish it from what it is not."—Hamilton, Vol. I.

87. "Energy can not exist except in connexion with matter. Hence, since in the space between the sun and the earth, the luminous and thermal radiations, which have left the sun and which have not reached the earth, possess energy, the amount of which per cubic mile can be measured, this energy must belong to matter existing in the interplanatory spaces, and since it is only by the light which reaches us that we become aware of the existence of the most remote stars, we conclude that the matter which transmits light is disseminated through the whole of the visible universe."—Maxwell's *Matter and Motion*, p. 93.

CHAPTER VIII.

Functions and Value of the Syllogism.

§ 1. According to Mill the syllogistic process is not the process according to which we reason. "All inference," says he, "is from particulars to particulars: general propositions are merely registers of such inferences already made, and short formulæ for making more. The major premiss of a syllogism consequently is a formula of this description; and the conclusion is not an inference drawn from the formula, but an inference drawn according to the formula; the real, logical antecedent or premiss being the particular facts from which the general proposition was collected by Induction[1]." "The value, therefore, of the syllogistic form, and of rules for using it correctly, does not consist in their being the form and the rules according to which our reasonings are necessarily, or even usually, made; but in their furnishing us with a mode in which these reasonings may always be represented, and which is admirably calculated, if they are inconclusive, to bring their inconclusiveness to light. An induction from particulars to generals, followed by a syllogistic process from those generals to other particulars, is a form in which we may always state our reasonings if we please. It is not a form in which we *must* reason, but it is a form in which we *may* reason, and into which it is indispensable to throw our reasoning, when there is any doubt of its validity: though when the case is familiar and little complicated, and there is no sus-

[1] *Logic*, Vol. I. p. 221.

picion of error, we may, and do, reason at once from the known particular cases to unknown cases [1]."

The universal type of the reasoning process, according to Mill, is as follows:—"Certain individuals have a given attribute; an individual or individuals resemble the former in certain other attributes; therefore they resemble them also in the given attribute [2]." This type is not, however, conclusive like the syllogism from the mere form of the expression; but must, in every case, be examined by the canons and rules of Induction. For example, 'all men now living resemble those men who have heretofore died' in certain attributes; whether from their resemblance in these attributes we may infer also their resemblance in the attribute 'mortality' is a question of Induction, and must be determined by its canons. If we may infer this attribute of 'all men now living,' we may infer it also of all other individuals that resemble the men who have died in the same attributes. This process of inference admits of a division into two steps: (1) "That of ascertaining what attributes are marks of mortality, universally, *i.e.*, under all circumstances, and (2) whether any given individuals possess those marks."

Conformably to usage, the first step or process, namely, that of establishing the general proposition, is called Induction, and the second step in "the reasoning operation, which is substantially that of interpreting the general propositions," is called Deduction by Mill. Every process by which any thing is inferred respecting an unobserved case, consists similarly of an Induction followed by a Deduction. According to Mill, the syllogism is thus merely a process by which the real or complete meaning of a general proposition established by Induction is made explicit, and by which the validity of a reasoning is tested. It is, in other words, an *interpreter* of the general proposition and a *test* of reasoning. Its rules and canons are merely cautions against false reasoning. They merely help us in interpreting correctly the true meaning of general propo-

[1] *Logic*, Vol. I. pp. 227—8. [2] *Ibid.* p. 232.

sitions, and in applying them to particular cases. In ordinary discourse the reasoning is never conducted nor stated in the syllogistic form; but whenever there is any doubt about its validity, we may, or rather we *must*, throw it into the syllogistic form, and if it admits of being so expressed, we may be perfectly sure of its being valid. The syllogistic is not, therefore, the process according to which we usually reason. The universal process of reasoning is, according to Mill, from some particulars to other particulars; and the syllogistic process is merely a *test* of the validity of this process.

§ 2. Nor, according to Mill, is the syllogistic mode of arguing a sound one. "For," says he, "it must be granted that in every syllogism, considered as an argument to prove the conclusion, there is a *petitio principii*. When we say, 'all men are mortal, Socrates is a man; therefore Socrates is mortal,' it is unanswerably urged by the adversaries of the syllogistic theory, that the proposition 'Socrates is mortal' is presupposed in the more general assumption 'All men are mortal'; that we cannot be assured of the mortality of all men, unless we are already certain of the mortality of every individual man, &c., &c.; that, in short, no reasonings from generals to particulars can as such prove anything; since from a general principle we can not infer any particulars but those which the principle itself assumes as known[1]."

Regarded as a mode of Probation, the syllogism involves, according to Mill, the fallacy of *petitio principii*, that is, the conclusion is presupposed by the major premiss. The proposition 'all men are mortal' can not be true, unless the conclusion 'Socrates is mortal' is true. The truth of the latter is presupposed by the former, or the former can not be true unless the latter is. When you have assumed the major, you have already taken for granted the conclusion. Thus the conclusion is not really proved by the premisses of the syllogism. It is, on the contrary, proved by those particular cases of observation which

[1] *Logic*, Vol. I. p. 210.

establish the general or major premiss. It is these that are alike the evidence of the major premiss and of the conclusion of the syllogism.

The syllogism is thus, according to Mill, neither the process according to which we reason, nor an argument which is sound and free from fallacy. Is it, then, altogether useless? No, says Mill, its proper function is to interpret a general proposition and apply it to particular cases, and its real value consists in being an infallible test of the validity of the true process of reasoning. This process is, according to Mill, from particulars to particulars in accordance with the laws and canons of Induction. But when an inference is drawn from some particulars to some other particulars, we can not be quite certain that the reasoning is valid unless it admits of being thrown into the syllogistic form. That is, if, from 'some particulars,' we can infer a general proposition, and if with this general as a major premiss, and with 'some other particulars' as a minor, we can form a valid syllogism, then the reasoning is valid. If the general can not be inferred, and the syllogism can not be formed, then the reasoning is invalid. For example, the reasoning that "all Kings now living are mortal, because all men in past ages have died," is completed according to inductive methods; but it will not be valid, unless a general proposition "all men are mortal" can be inferred from the particular cases of men who have died in past ages, and unless 'all kings now living' are really referable to the class 'man,' that is, the validity of the reasoning which is actually and really conducted from particulars to particulars in accordance with the canons of Induction, may be tested by reducing it to the following syllogism: "all men are mortal, all kings now living are men; therefore all kings now living are mortal."

This view of the functions and value of the syllogism, first propounded by Mill, has been adopted by Sir John Herschel, Dr Whewell, Mr Bailey, Professor Bain, and others. It has, on the other hand, been strongly opposed by Mansel, Professor De Morgan, Dr James Martineau, and others.

§ 3. There are two essential points in Mill's view of the syllogism,—(1) that it is not the usual process of reasoning, (2) that it involves the fallacy of *petitio principii*.

On the first point Mill maintains, that the universal process of reasoning is from particulars to particulars; and on the second point, that the real proof of the conclusion is not the premisses of the syllogism, but the facts of observation and testimony on which the major premiss itself is founded. On these two points the following observations may be made:—

1. It is true that the syllogism is not the process by which we usually reason. But it is equally true that our usual reasonings will not be valid, and therefore not deserve the name, unless they are capable of being reduced to the syllogistic form. Mill seems to make a confusion between the business of Psychology and that of Logic. It is not the business of the latter to give an account of the various processes by which people reason correctly or incorrectly, but to give an account of the processes by which they *ought* to reason, and *must* reason if they wish to reason correctly. The former is the business of the Psychology of Reasoning, while the latter is the business of the Logic of Reasoning. Mill confuses these two, and makes both the business of Logic. Recognizing the distinction here drawn, it may be said that the syllogism is the type of all valid reasoning; for no reasoning will be valid, as Mill also allows, unless it can be thrown into the form of a syllogism. As a matter of fact, in daily life, men draw inferences in many different ways, but only those among them will be valid, and properly deserving of the name, which are capable of being ultimately reduced to the syllogistic form, the rest being nothing but suggestions of association, fancy, imagination, &c., wrongly called inferences[1].

§ 4. 2. Secondly,—Does the syllogism involve the fallacy of *petitio principii?* On this most important subject the following noteworthy remark by Dr James Martineau is well deserving of being quoted; and as the book in which it is con-

[1] *Vide* Appendix D.

tained is not usually accessible to students, I will give it in full :—

"From the embarrassment of this objection we may extricate ourselves at once by simply remembering that, in the nature of things, or in the sight of a perfect intellect, whose processes are unconscious of succession or delay, *all* reasoning must involve a *petitio principii*, the conclusion being already discerned on the first announcement of the premiss. Ratiocination itself becomes nugatory in presence of a mind seeing by intuition what others reach by sequence. As soon as we descend to a more tardy and limited intelligence, there will be some beliefs that are mediately reached : the same truths which to one being are contained within their *arche* (ἀρχή) are seen by another lying at some distance from it. The *petitio principii* is thus entirely relative to the state and range of the individual understanding, and cannot be established as a fault against an argument by merely showing that the inference *might* be thought already in the assumption, but only by showing that it *must* be. If Mr Bailey can convince us that it is impossible to conceive the proposition 'birds are warm-blooded' without simultaneously contemplating the particular case of the swallow, we will grant that the conclusion 'swallows are warm-blooded' is a new inference of *idem per idem*. But if not,—if the general law can be formed, and, as he allows, rationally formed, without the mind having ever encountered this special instance,—it is vain to pretend that the conclusion only repeats in part the thought contained in the premiss. This is, no doubt, true of the reasoner, who, to bring conviction, invents the syllogism in question : he selects his general rule precisely, *because* he foresees what it contains ; but in using it, he assumes in his learners a different state of mind,—in which the law has been apprehended and the example has been missed. Whenever a teacher and a learner are engaged together, the arguments comprehended in the didactic process involve a *petitio principii* to the former, but not to the latter. Upon this difference, the consciousness in one man, the unconsciousness in another, of what, according to the laws of

thought, a given proposition may imply, depends persuasion. Mr Mill, we are aware, treats this doctrine with no respect, and calls Archbishop Whately to severe account for sanctioning it. 'When you admitted the major premiss,' contends Mr Mill, 'you asserted the conclusion; but, says Archbishop Whately, you asserted it by implication merely: this, however, can here only mean that you asserted it unconsciously; that you did not know you were asserting it; but if so, the difficulty revives in this shape,—Ought you not to have known? Were you warranted in asserting the general proposition without having satisfied yourself of the truth of everything which it fairly includes? And if not, what then is the syllogistic art but a contrivance for catching you in a trap and holding you fast in it?' Mill's *Logic*, Vol. I. p. 212. This is a clever scolding, no doubt; but, as it seems to us, indifferent logic. The phraseology itself is highly objectionable. In order to make out that the conclusion is anticipated in the premisses, though not foreseen by the reasoner, Mr Mill resorts to a doctrine of '*unconscious assertion*' which we can only compare with the hidden sense of prophecy imagined by divines. 'Assertion' not being an automatic articulation by the lips, but a mental act, the intentional predication of a certain attribute present in thought respecting a certain subject also present in thought can not be 'unconscious'; and the epithet does but evade the fact that the assertion in question is not there at all. To another mind, indeed, and to the same mind at a future time, the proposition may suggest the application which the sentence as uttered did not contemplate: but these are phenomena foreign to the immediate act of predication, and not entitled to be imported into its description. And as to Mr Mill's demand that no general proposition shall be uttered till the speaker holds in his thought all instances to which it may be applied, we know of nothing more simply impossible or more entirely destructive of all scientific method whatever. The foresight of its particular cases is *not* 'fairly included' in the meaning or in the evidence of a general rule; and a person may reasonably assent to the law of refraction without any suspicion

of the vast compass of facts over which its interpretation ranges. There are grounds,—whatever account we may give of them,—for ascribing attributes to certain *natures* or *kinds* of being, without going through the objects included under them or having any prescience of their actual contents. It is not necessary to know the natural history of all the varieties of mankind before we can venture to affirm mortality of human beings in general. To revert to our old syllogism: 'All birds are warm-blooded, swallows are birds; therefore swallows are warm-blooded.' It is surely possible (1) to think the attribute 'warm-blood' of the genus (bird) without thinking it of the species (swallow),—that is, to have the *major* premiss without the conclusion; (2) to ascribe to the species (swallow) the nature of the genus (bird) without therewith ascribing to it all the concomitants (as warm-blood) of the genus,—that is, to have the *minor* premiss without the conclusion. But it is *not* possible to do both these things without at once recognizing the conclusion. This is all that is required by the theory of the syllogism; and against this Mr Mill can only urge, that *if* it be true,—why, it *ought not* to be true[1]."

According to Dr Martineau, therefore, the syllogism does not involve the fallacy of *petitio principii*,—(1) because the conclusion is not present in thought while the major or the minor premiss is, and (2) because the conclusion does not follow from the major alone, nor from the minor alone, but from the two taken jointly. The second point is quite self-evident and follows from the definition of a syllogism. The conclusion does not follow from either of the premisses singly, but from both of them taken jointly. Professor De Morgan[2] and a writer in the *British Quarterly Review* (August, 1846) also point out that Mill's view makes the minor proposition of a syllogism quite superfluous and unnecessary, and that as the minor premiss is an essential part of the syllogistic argument, the conclusion not being deducible from the major premiss alone, Mill's objection to the syllogism is

[1] *Essays*, Vol. II. pp. 336—339.

[2] *Formal Logic*, pp. 257—259.

quite untenable. "The whole objection," says De Morgan, "tacitly assumes the superfluity of the minor, that is, tacitly assumes we know Plato to be a man as soon as we know him to be Plato[1]." The reviewer says that if the major premiss included the conclusion, "we should be able to affirm the conclusion without the intervention of the minor premiss; but every one sees that that is impossible." No general proposition can be applied to a new case, unless a minor proposition affirms the new case to come under the general or to possess the marks characteristic of the subject of the general.

In reply to the first point Mill would of course say that though the conclusion is not present in thought, it ought to have been, that no one ought to admit the major without seeing that he thereby also admits the conclusion. Martineau admits that all this is actually seen by the teacher, but that it is not seen by the learner. Hence what may be a *petitio principii* to the former is not so to the latter. The value of an argument depends on the state of the mind to which it is addressed. To the omniscient mind all reasoning must involve a *petitio principii*. To us what is a *petitio principii* at one time was not so at another. If we can *somehow get a general proposition* without actually thinking of, or observing, all the particulars to which it is applicable, then the syllogism can not reasonably be said to be guilty of the charge of a *petitio principii*. "There are," says Martineau, "grounds,—whatever account we may give of them,—for ascribing attributes to certain *natures* or *kinds* of being, without going through the objects included under them or having any prescience of their actual contents." This is the question of questions. Can we ascribe attributes to certain *natures* or *kinds* of being, without having examined *all* the particular objects included under them? In other words, can we establish the truth of a universal proposition from the truth of certain cases included in it, without examining all the possible cases? This is the great problem of Inductive Logic. It is the business of Inductive

[1] *Formal Logic*, p. 259.

Logic to lay down rules and canons, to which we must conform, in order that we may infer general or universal propositions from particular ones. Deductive Logic takes for granted that there *are* universal propositions, whatever account may be given of their origin, nature, and grounds by philosophers of different schools. If there are such propositions, the syllogism can not reasonably be regarded as a *petitio principii;* it becomes, on the contrary, a very useful and sound process of reasoning. If it can be quite satisfactorily established, for example, by the rules and canons of Induction from the observation of *some* cases, that the attribute B is a mark of A,—that wherever B is, A is; and if in a new case C, I find the attribute B, I can reasonably infer the attribute A, of which the former is, by supposition, an unfailing mark. This reasoning, when fully expressed, gives rise to the following syllogism "All B is A, C is B, therefore C is A." It may be also thus stated, "A co-exists with B, B co-exists with C, therefore A co-exists with C[1]." Here, in establishing the major premiss, the *new* case in question was not in any way concerned. It had in fact no existence at all, real or imaginary, and therefore could not be known, or thought of, when the major was established. You may of course doubt the truth of the major premiss, or that the new case in question has the attribute B; but granting both the premisses to be true, you can not doubt the conclusion,—you must regard it as certain. And this brings us to the question of the proper nature of Deductive Inference.

§ 5. *Hypothetically necessary* character of all Deductive Inference. In deductive or syllogistic reasoning we draw conclusions from given propositions as data. Given the premisses, we infer the conclusion that follows *necessarily* from them. We are not in any way concerned to prove our premisses; but our conclusion must be true, if the premisses be true. Hence it is evident that the truth we arrive at by deductive or syllogistic reasoning is entirely of a hypothetical character, depending for

[1] *Vide* Appendix A, pp. 282—284.

its trustworthiness entirely on the trustworthiness of the data. If the latter be true, the former must be so. The premisses of a syllogism, though they may be immediately the conclusions of prior syllogisms, are ultimately the results of Induction, Observation, Perception, or Intuition; but whatever their origin may be, Deductive Logic has nothing to do with it. All that it is concerned with is, the legitimacy of the conclusion or conclusions that are drawn from the premiss or premisses. To its student Deductive Logic offers the following wholesome advice: "If you wish to live happily in my domain, obey my Laws. If you desire to enjoy the peace of certitude, conform to the rules and conditions I have laid down. I take no account of your prejudices, passions, instincts, habits, associations, interests, and tendencies, which may induce you to infer any thing from any thing else: you must, under all circumstances, implicitly or explicitly obey my Laws, if you desire to attain your object. If you reason from some to all, you reason against my express Law, and though your conclusions may in some cases be accidentally true, the means you employ to attain your end are none the less unlawful. If you reason from particulars to particulars, you do this at your own risk and responsibility. The Law which I lay down is that you infer the particular from the general, or the less general from the more general, and not conversely."

CHAPTER IX.

Probable Reasoning and Probability.

§ 1. If both the premisses of a syllogism are necessary, or assertory, or probable, the conclusion is necessary, assertory, or probable. If the modality of one premiss be different from that of the other, the conclusion has the less certain modality. For example, 'B must be A, C is B: ∴ C is A'; 'B is A, C is probably B: ∴ C is probably A.' Now what is the meaning of the propositions 'C is probably B' and 'C is probably A'? From the two premisses 'A is probably B' and 'B is probably C,' we may infer 'A is probably C.' Is this inference always legitimate? Is the meaning of *probably*, or rather is the *degree of probability*, the same in the conclusion as in either of the premisses? Under what conditions is the conclusion valid? In order to answer these questions, we must first of all state the meaning of a Probable Proposition.

§ 2. The Meaning of a Probable Proposition.

'It will *probably* rain to-morrow,' or 'He will *probably* die,' means, *subjectively*, that my belief in the event in question is not full or complete, is of a degree less than the highest; and *objectively*, that the evidence for the happening of the event in question is not of such a nature as to make it a certainty. That this is the meaning of the proposition will be evident if we consider the meaning in the assertory form. 'It rains,' 'He is dead,' 'The sun rises,' 'Fire is burning': in each of these my belief is of the highest degree, and the event in ques-

tion is quite certain : *subjectively*, there is no trace of doubt, and *objectively*, there is not the least uncertainty about the event. When the word *probably* is added to the copula, the proposition means, *subjectively*, that the state of my mind in regard to the event is a mixture of belief and doubt, partial belief caused by certain evidence for, and partial doubt caused by certain evidence against, the event, that is, a state of incomplete belief caused by incomplete evidence for the event; and it means, *objectively*, that there is some evidence for, and some against, the event, or at any rate that all the evidence attainable is not such as to make the event a certainty. For example, 'He will probably die' means that there are certain appearances that are symptoms of death, and that there are others which are not: that there are certain signs or marks from which we may infer that death will result, and that there are others from which we may infer the contrary; so that altogether the evidence is conflicting, and the state of mind resulting may be said to be a state of partial belief, or a mixture of belief and doubt.

In this sense the words 'probably,' 'probable,' 'probability' mean any degree of belief less than the highest, and any evidence for the event less than certainty. If we represent full belief and highest certainty by 1, we may represent different degrees of 'probability' by fractions such as $\frac{4}{5}$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. In ordinary language the word 'probable' means 'more likely than not,' and in this sense 'probability' would always be represented by fractions greater than $\frac{1}{2}$. But, in the widest sense in which it is used here, it may be represented by any fraction however small or large, and corresponds exactly to the mathematical word 'chance.'

The probability of a proposition may, then, be represented by a fraction. But what is the exact meaning of the fraction, and how do we get it? The meaning of the proposition 'It will probably rain to-morrow' is, we may say, that the probability of its raining to-morrow is $\frac{m}{n}$; or the meaning of the proposition 'He will probably die this year' is that the probability

of his dying is $\frac{2}{3}$, or $\frac{1}{3}$, or any other fraction. Now, how is this fraction obtained, and what is its real meaning? We cannot discuss this question here. We shall adopt the view held by Dr Venn, which appears to be the best and most reasonable. "I consider," says he, "that these terms (probability, chance) presuppose a series; within the indefinitely numerous class which composes a series, a smaller class is distinguished by the presence or absence of some attribute or attributes. * * * * These larger and smaller classes respectively are commonly spoken of as instances of the 'event,' and of 'its happening in a given particular way.' Adopting this phraseology, which, with proper explanations, is suitable enough, we may define the probability or chance (the terms are here regarded as synonymous) of the event happening in that particular way as the numerical fraction which represents the proportion between the two different classes in the long run. Thus, for example, let the probability be that of a given infant living to be 80 years of age. The larger series will compose all men, the smaller all who live to 80. Let the proportion of the former to the latter be 9 to 1; in other words, suppose that 1 infant in 10 lives to 80. Then the chance or probability that any given infant will live to 80 is the numerical fraction $\frac{1}{10}$[1]." Conversely, if the probability of a man living to 80 be $\frac{1}{10}$, this implies that in every 10 persons one only lives to that age. Similarly, if the probability of its raining to-morrow be $\frac{2}{3}$, this implies that in every three cases like the present, rain happens in two cases on the following day. If the probability of a man's dying of a certain disease be $\frac{1}{3}$, it means that in every three cases of that disease one dies. The two classes, one larger and the other smaller, the proportions between which constitute the probability are, in the last example, (1) the class of persons who have had that disease, and (2) the special class within the other of persons who have died of it; and the proportion of the second to the first is represented by the fraction $\frac{1}{3}$.

[1] Venn's *Logic of Chance*, 2nd ed., p. 145.

§ 3. The Rules of Immediate Inference.

Every probable proposition is thus connected with what Dr Venn aptly calls a *Proportional* proposition of the form 'm A's in n are B.' It can be shown that every probable proposition must ultimately be traced to a proportional proposition of that form, and that, without tracing it to such a proposition, we can give no rational account of its meaning, when the probability is represented by a fraction. A *proportional* proposition is to be distinguished from a *universal* of the form 'All A is B.' From the latter we may infer that 'Any A or sub-class of A is B.' From the former we may infer that 'Any A is *probably* B,' the probability being represented by the fraction $\frac{m}{n}$. Given that 9 men in 10 of any assigned age live to 40, we may immediately infer that the probability of a man of that age living to 40 is $\frac{9}{10}$. Given that 3 in 4 men in India are Hindus, we may immediately infer that the probability of a man in India being a Hindu is $\frac{3}{4}$. Given that 2 in 4 candidates will pass at the examination, we may immediately infer that the probability of a candidate's passing is $\frac{1}{2}$. Thus, from every proportional proposition, we may infer a probable one, the probability of which is represented by a fraction. Conversely, from a probable proposition we may infer a proportional one. Given the probable proposition 'A is probably B,' the probability of which is represented by the fraction $\frac{2}{3}$, we may infer the proportional proposition '2 in 3 A's are B.' Given that the probability of a man under certain circumstances becoming rich is $\frac{1}{10}$, we can immediately infer that 1 man in 10 under the same circumstances becomes rich. Given that the probability of an event happening is $\frac{3}{5}$, we can infer that 3 events in 5 of that nature do usually happen.

Examples.

'Most A's are B': from this we can infer that the probability of any A being B is greater than $\frac{1}{2}$.

'$\frac{3}{4}$ of A are B' or '3 A's in 4 are B': from this we can infer that the probability of any A being B is $\frac{3}{4}$.

'Some A's are B': from this we can infer nothing. Similarly from 'Many A's are B' we can infer nothing. The proportion of A that is B must be stated before we can infer the probability of any A being B., *i.e.*, the proposition must be a proportional one.

'The greater number of A are B': from this we can infer that the probability of any A being B is greater than $\frac{1}{2}$.

The same inference follows from the proposition 'the majority of A are B.'

§ 4. The Rules of Mediate Inference.

The rules of mediate inference in probability may be divided into two classes—(1) those which are *formal*, and (2) those which are more or less *experimental*. The former follow necessarily from probable propositions by the mere application of arithmetic; whilst the latter "either depend upon peculiar hypotheses, or demand for their establishment continually renewed appeals to experience and extension by the aid of the various resources of Induction."

§ 5. (1) The Formal Rules of Mediate Inference.

"The fundamental rules of probability strictly so called, that is, the *formal* rules, may be divided into two sub-classes—(i) those obtained by addition or subtraction on the one hand corresponding to what are generally termed the connection of *exclusive or incompatible* events, and (ii) those obtained by multiplication or division on the other hand corresponding to what are commonly termed *dependent* events[1]."

(i) Rules of *Exclusive* Events.—"If the chances of two exclusive or incompatible events be respectively $\frac{1}{m}$ and $\frac{1}{n}$, the chance of one or other of them happening will be $\frac{1}{m}+\frac{1}{n}=\frac{m+n}{mn}$. Similarly, if there were more than two events of the kind in question." A bag, for example, contains 16 balls, of which 10 are red and 6 are blue. That is, 10 balls in 16 being red, and 6 in 16 being blue, the chances of drawing a red and a blue ball

[1] Venn's *Logic of Chance*, 2nd ed., p. 150.

are respectively $\frac{10}{16}$ and $\frac{6}{16}$. Therefore the chance of drawing either is the sum of $\frac{10}{16}+\frac{6}{16}=1$; that is, the ball drawn is certain to be a red or a blue ball and can not be anything else. The events here are exclusive or incompatible, because while one happens the other can not; when a red ball, for instance, is drawn, a blue ball can not be drawn at the same time. I may of course draw two balls one after another, but, while drawing once, one ball must be drawn, and it must be either red or blue. Suppose the ball first drawn is a red one, and is not replaced in the bag; then there are now 9 red and 6 blue balls in the bag, and the chances respectively are $\frac{9}{15}$ and $\frac{6}{15}$. Suppose at the second drawing a blue ball is drawn; now there are 9 red and 5 blue balls in the bag, and the chances respectively are $\frac{9}{14}$ and $\frac{5}{14}$. Suppose on drawing a third time a red ball comes out; now there are in the bag 8 red and 5 blue balls, and the chances respectively are $\frac{8}{13}$ and $\frac{5}{13}$.

The following is a sort of corollary to the above:—

"If the chance of one or other of two incompatible events be $\frac{1}{m}$ and the chance of one alone be $\frac{1}{n}$, the chance of the remaining one will be $\frac{1}{m}-\frac{1}{n}=\frac{n-m}{mn}$. For example, if the chance of any one dying in a year is $\frac{1}{10}$, and his chance of dying of some particular disease is $\frac{1}{100}$, his chance of dying of any other disease is $\frac{9}{100}$.[1]" In the example given above, the chance of drawing a red or a blue ball is 1, and the chance of drawing a blue ball is $\frac{6}{16}$, therefore the chance of drawing a red ball is $1-\frac{6}{16}=\frac{10}{16}=\frac{5}{8}$.

(ii) Rules of *Dependent* Events.—"We can also make inferences by multiplication or division. Suppose that the two events instead of being incompatible are connected together in the sense that one is contingent upon the occurrence of the other. Let us be told that a given proportion of the members of the class or series possess a certain property, and a given proportion again of these possess another property, then the proportion of the whole

[1] Venn's *Logic of Chance*, p. 152.

which possess both properties will be found by multiplying together the two fractions which represent the above two proportions. Of the inhabitants of London, twenty-five in a thousand, say, will die in the course of the year; we suppose it to be known also that one death in five is due to fever; we should then infer that one in 200 of the inhabitants will die of fever in the course of the year. It would, of course, be equally simple by division to make a sort of converse inference. Given the total mortality *per cent.* of the population from fever, and the proportion of fever cases to the aggregate of other cases of mortality, we might have inferred, by dividing one fraction by the other, what was the total mortality *per cent.* from all cases.

"The rule, as given above, is variously expressed in the language of probability. Perhaps the simplest and best statement is that it gives us the rule of dependent events, that is, if the chance of one event is $\frac{1}{m}$, and the chance that if it happens another will also happen is $\frac{1}{n}$, then the chance of the latter is $\frac{1}{mn}$. In this case it is assumed that the latter is so entirely dependent upon the former that, though it does not always happen with it, it certainly will not happen without it; the necessity of this assumption, however, may be obviated by saying that what we are speaking of in the latter case is the *joint* event, viz., both together if they are simultaneous events, or the latter *in consequence of* the former, if they are successive[1]."

Examples of (ii).

Suppose the chance of a boy of 10 years living to 20 is $\frac{2}{3}$, and if he lives to that age the chance of his being educated is $\frac{1}{3}$, then the chance of his being educated is $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$. That is, 2 boys in 9 of the age of 10 years live to 20 and become educated.

Suppose the chance of there being plenty of rain this season is $\frac{3}{4}$, and the chance of the crops growing $\frac{3}{8}$, if the former event happens;

[1] Venn's *Logic of Chance*, pp. 153—4.

then the chance of there being rain and of the crops growing, *i. e.*, of the *joint* event, is $\frac{3}{4} \times \frac{5}{6} = \frac{5}{8}$. This is in fact the chance of the last event, which happens *in consequence* of the first.

Suppose the chance *of* a person's acting prudently under certain circumstances is $\frac{2}{3}$, and if he acts prudently, the chance of his being happy is $\frac{5}{6}$, then the chance of his both acting prudently and being happy, or, in other words, of his being happy which happens *in consequence of* the first event, is $\frac{2}{3} \times \frac{5}{6} = \frac{10}{18} = \frac{5}{9}$. As the second event depends in all these cases upon the first, as the happening of the one is dependent upon the happening of the other, the two events are called *Dependent* or *Contingent* events, and should be distinguished, on the one hand, from incompatible events, and, on the other, from independent events.

Similarly, if the chance of A being B is $\frac{3}{4}$, and if, this happening, the chance of B being C is $\frac{1}{3}$, then the chance of A being C is $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$. That is, 1 A in 4 is C.

Here we may take up the example given at the beginning of this chapter, and state the condition under which the inference is valid so far as we are able to do at present:—

$$\text{A is probably B} \left(\text{probability} = \frac{1}{m}\right),$$

$$\text{B is probably C} \left(\quad ,, \quad = \frac{1}{n}\right);$$

$$\therefore \text{A is probably C} \left(\quad ,, \quad = \frac{1}{mn}\right).$$

Here the probability of the conclusion will be the product of the probabilities of the premisses, if they are dependent events in the sense explained above. That is, the reasoning will be valid if it admits of being stated as follows:—"The chance of A being B is $\frac{1}{m}$, and, this happening, the chance of B being C is $\frac{1}{n}$, then the chance of A being C is $\frac{1}{m} \times \frac{1}{n} = \frac{1}{mn}$"; or as follows: "One A in m is B, and, this happening, one B in n is C, therefore one A in m × n is C"; or as follows: "A is probably B

$\left(\text{probability} = \frac{1}{m}\right)$, whatever A is B is probably C $\left(\text{probability} = \frac{1}{n}\right)$; therefore A is probably C $\left(\text{probability} = \frac{1}{mn}\right)$." These three statements express in different ways the same matter of fact.

§ 6. (2) The *Experimental* Rules of Mediate Inference.

The rules of this class "stand upon a somewhat different footing from the above in respect of their cogency and freedom from appeal to experience or to hypothesis. In the first class, we considered cases in which the data were supposed to be given under the condition that the propositions which distinguish the different kinds of events, whose frequency was discussed, were respectively known to be disconnected and known to be connected. Let us now suppose that no such conditions are given to us. One man in 10, say, has black hair, and 1 in 12 is short-sighted, what conclusion could we then draw as to the chance of any given man having one only of these two attributes, or neither, or both? It is clearly possible that the properties in question might be inconsistent with one another, so as never to be found combined in the same person, or all the short-eyed might have black hair, or the properties might be allotted in almost any other proportion whatever, except as restricted by the arithmetical conditions. If we are perfectly ignorant upon these points, it would seem that no ifferences whatever could be drawn about the required chances[1]." If, on the other hand, we are warranted in making the assumption that "the division into classes caused by each of the above distinctions should subdivide each of the classes created by the other distinction in the same ratio in which it subdivides the whole," then the following rule of inference will hold good:—

"If the chances of a thing being p and q are respectively $\frac{1}{m}$ and $\frac{1}{n}$, then the chance of its being both p and q is $\frac{1}{mn}$, the

[1] Venn's *Logic of Chance*, pp. 154—5.

chance of its [redacted] and not q is $\frac{n-1}{mn}$, and the chance of its being not p and not q is $\frac{(m-1)(n-1)}{mn}$, where p and q are independent. The sum of these chances is obviously unity, as it ought to be, since one or other of the four alternatives must necessarily exist." This is the rule of the so-called *independent* events, the nature of the independence being defined by the supposition stated above.

Taking the instance mentioned above, "let us take a batch of 1,200 as a sample of the whole. Now, from the data which were originally given to us, it will easily be seen that in every such batch there will be on the average 120 who have black hair, and therefore 1,080 who have not. And here in strict right we ought to stop, at least until we have appealed again to experience; but we do not stop here. From data which we assume," that is, from the data which follow from granting the assumption stated above to be true, "we go on to infer that of the 120, 10 (*i.e.* $\frac{1}{12}$ of 120) will be short-sighted, and 110 (the remainder) will not. Similarly we infer that of the 1,080 90 are short-sighted, and 990 are not. On the whole, then, the 1,200 are thus divided:—Black-haired, short-sighted, 10; short-sighted without black hair, 90; black-haired men who are not short-sighted, 110; men who are neither short-sighted nor black-haired, 990." If that assumption had not been true, we should not have been justified in drawing those inferences, for of the 1,200 there would be 120 black-haired; and the 100 short-sighted might be none of the 120 who had black hair, and so forth. The necessary and sufficient condition of our inferences being valid is that of the 120 who have black hair, 10 must be short sighted, as also the same proportion of the 1,080 who have not black hair; and that taking likewise the short-sighted first, of the 100 who are short-sighted, 10 must have black hair as well as the same proportion of the 1,100 who are not short-sighted. That is, the condition which is assumed to be true is, that the division into classes caused by each of the given distinctions should subdivide each of

the classes created by the other distinctions in the same ratio in which it subdivides the whole. This condition being true, the rule of inference given above is quite correct and free from all objection. In the form in which it is usually given it is open to objection, and leads to inferences which are not formally valid, the events being assumed to be independent where nothing is known about the distribution of the properties. But we have seen that it is necessary that we should possess some positive knowledge of the distribution before we can apply the rule.

We can now further state the *condition* or *supposition* under which the inference is valid in the case of our original example:—

$$\text{A is probably B} \left(\text{probability} = \frac{1}{m}\right),$$

$$\text{B is probably C} \left(\quad \text{,,} \quad = \frac{1}{n}\right);$$

$$\therefore \text{A is probably C} \left(\quad \text{,,} \quad = \frac{1}{mn}\right).$$

The probability of the conclusion will be the product of the probabilities of the two premisses, *if the subdivision of A created by B be subdivided in the same ratio in which the whole of B is subdivided by C.* For example, suppose the probabilities to be $\frac{2}{3}$ and $\frac{1}{3}$ respectively, and A to be represented by a sample of 36; then, according to the first premiss, 24 A's in 36 are B, and according to the second premiss, and the condition assumed, 8 B's in these 24 are C: therefore, 8 A's in 36 are C,—that is, the probability of A being C is $\frac{8}{36}$, or $\frac{2}{9}$, which is equal to the product of the probabilities of the two premisses.

In certain cases, however, it is possible to draw valid inferences without any assumption whatever—I mean those cases in which the sum of the probabilities of the two *independent* events exceeds unity, and in which the two premisses are as in the third syllogistic figure. The rule of inference in such cases is as follows:—if the chances of a thing being p and q are, respectively, $\frac{1}{m}$ and $\frac{1}{n}$, then the chance of its being both p and q

is $\frac{1}{m} + \frac{1}{n} - 1$, [redacted] chance of its being p and not-q is $\frac{1}{m} - \frac{1}{n}$, if $\frac{1}{m}$ be greater than $\frac{1}{n}$, where p and q are independent. For example, 3 A's in 4 are B, and 1 A in 3 is C; therefore 1 A in 12 must be both B and C,—that is, the probability of A being B is $\frac{3}{4}$, and the probability of A being C is $\frac{1}{3}$, therefore the probability, according to the given rule, of A being both B and C is $\frac{3}{4} + \frac{1}{3} - 1$, or $\frac{1}{12}$. Let A be 24 men, of whom $\frac{3}{4}$, that is 18, are educated, B, and $\frac{1}{3}$, that is 8, are rich, C, then $\frac{3}{4} + \frac{1}{3} - 1$, or $\frac{1}{12}$, that is 2, must be both educated and rich, and $\frac{3}{4} - \frac{1}{3}$, or $\frac{5}{12}$, that is 10, both educated and not rich. Similarly, if $\frac{4}{5}$ of A are B and if $\frac{3}{5}$ of A are C, then $\frac{2}{5}$ of A must be both B and C, and $\frac{1}{5}$ of A both B and not C. From the first conclusion it follows that some B's are C, and some C's are B.

§ 7. Exercises.

1. Fully explain the meaning of the following propositions, and draw the inferences which follow from them:—

(*a*) The substance A is probably a metal.
(*b*) B is probably a prudent man.
(*c*) D will probably pass at the F. A. Examination.
(*d*) E will probably live to the age of eighty.
(*e*) The sun will most probably rise to-morrow.
(*f*) All virtuous men are probably happy.
(*g*) This fossil is probably carboniferous.
(*h*) The luminiferous ether probably gravitates.

2. The probability of a fossil being mesozoic is $\frac{1}{3}$; and if it is mesozoic, the probability of its being cretaceous is $\frac{1}{4}$; and if it is cretaceous, the probability of its being found in the English chalk formation is $\frac{7}{8}$. Calculate the probability of the fossil being found in the English chalk.

3. The probability of a new-born child living to the age of 25 years is $\frac{1}{2}$; and if it lives to that age, the probability of its being well-educated is $\frac{1}{4}$; and if it is well-educated, the probability of its being a distinguished person is $\frac{1}{20}$. Calculate the probability of the new-born child being a distinguished person.

4. Three elements in four are conductors of electricity; four in five are conductors of heat: draw the conclusions which follow if the total number of elements be sixty.

5. The population of London is 4,000,000; and there are 500 distinguished men in all professions, suppose 100 in the medical, 150 in the literary, 100 in the army and navy, 50 in the church, 25 great orators, 25 engineers, 50 scientific men; what is the probability of a Londoner being a distinguished man, and belonging to any of these professions?

6. Test the following:—(1) Most A's are B; most B's are C: therefore some A is C. (2) Most A's are B; most A's are C: therefore some B is C.

7. The probability of a man's dying within two years of fever is $\frac{1}{8}$, of cholera $\frac{3}{8}$, of consumption $\frac{1}{4}$. What are the probabilities of his dying of fever or consumption, of cholera or consumption, and of fever or cholera?

8. Thirty *per cent.* of fishes are edible, and twenty *per cent.* are freshwater. Calculate the probabilities of a fish being edible, freshwater, edible and freshwater, edible and not-freshwater, freshwater and not-edible, and not-edible and not-freshwater.

APPENDIX.

A.—Canons or Axioms of the Syllogism according to Logicians.

§ 1. *Lambert's Canons for the so-called Imperfect Figures.*—In opposition to the view that all the figures except the first are imperfect, because they have no canons of their own like the '*Dictum de Omni et Nullo*' for the first or *perfect* figure, and that, therefore, syllogisms in those figures must be reduced to the first, Lambert (in his *Neues Organon*, Leipzig, 1764) enunciates a distinct canon for each figure, and thus places them all on an equality. For the first figure Lambert recognizes the 'dictum de omni et nullo' as usual. For the second figure he lays down a canon called '*Dictum de Diverso*,' which is as follows :—"If one term be contained in, and another excluded from, a third term, they are mutually excluded." This dictum is as self-evident as the 'dictum de omni et nullo.' On applying it to the sixteen possible combinations of premisses it will be found that the same valid moods are obtained as on any other method. It holds good in the moods *Cesare, Camestres, Festino, and Baroko.* In *Cesare* the term 'C' (taking A, B, and C as standing for the major, middle, and minor terms respectively) is included in 'B' in the minor premiss, and in the major premiss the term 'A' is excluded from 'B'; therefore, according to the 'dictum de diverso,' 'C' and 'A' are excluded from each other, that is, the conclusion is 'No C is A.' In *Baroko* the term 'A' is included

in 'B' in the major premiss, and the term 'some C' is excluded from 'B' in the minor premiss; therefore, according to the same dictum, 'Some C' and 'A' are excluded from each other, that is, the conclusion is 'Some C is not A.' The 'dictum de diverso' is similarly applicable to *Camestres* and *Festino*, and thus distinguishes the valid from the invalid moods in the second figure.

For the third figure Lambert enunciates the following canon, which is called *'Dictum de Exemplo'*:—"Two terms which contain a common part, partly agree, or if one term contains a part which the other does not, they partly differ." This is also self-evident, and may be easily applied to syllogisms in the third figure. In the valid mood *Darapti* of this figure 'B' is a part of 'A' in the major premiss, and also a part of 'C' in the minor premiss, that is, 'A' and 'C' have a common part 'B'; therefore they partly agree, that is, 'Some C is A,' according to the first part of the 'dictum de exemplo.' In the mood *Felapton* of the same figure the term 'C' contains 'B' in the minor premiss, while 'B' is not contained in 'A,' according to the major premiss; therefore 'C' and 'A' partly differ, that is, 'Some C is not A,' according to the second part of the same dictum. The first part of the 'dictum de exemplo' is similarly applicable to the other affirmative moods, and the second part to the other negative moods; and thus it distinguishes the valid from the invalid moods in the third figure.

For the fourth figure Lambert gives a canon called *'Dictum de Reciproco,'* which is stated as follows[1]:—"If no M is B, no B is this or that M; if C is or is not this or that B, there are B's which are or are not C." But it may be more clearly stated thus: If a term be included in a second term which is excluded from a third, then the third is excluded from the first; if a term be included in (or excluded from) a second term which is included in a third, then a part of the third is included in (or

[1] *Vide* Mansel's *Aldrich* (1849), p. 80; Hamilton's *Lectures*, Vol. IV. p. 441; and Ueberweg's *Logic*, p. 372.

excluded from [redacted]st. The first part is applicable to the mood *Camenes*, while the second part is applicable to the moods *Bramantip*, *Dimaris*, *Fesapo*, and *Fresison* in the fourth figure. Both parts of the dictum are self-evident, and require no explanation.

Lambert not only abolishes Reduction, and gives a canon for each of the so-called imperfect figures, but he also establishes their independence of the first figure and their equality with it, by showing that each figure is by its nature especially adapted for a particular kind of argument, and that we naturally think and express our thoughts in certain cases in one figure rather than in another. "For example, the proposition, *Some stones attract iron*, everyone will admit, because *The magnet is a stone* and *attracts iron*. This syllogism is in the third figure. In the first, by conversion of one of its premisses, it would run thus:—

(A)	All magnets attract iron	...	(major premiss),
(I)	Some stones are magnets	...	(minor premiss);
(I) ∴	Some stones attract iron	...	(conclusion).

"Here we are unaccustomed to the minor proposition, while it appears as if we must have all stones under review, in order to pick out magnets from among them. On the other hand, that *the magnet is a stone* is a proposition which far more naturally suggests itself, and demands no consideration. In like manner:—*A circle is no square;—for the circle is round,—the square not.* This proof (in the second figure) is as follows, when cast in the first:—

What is not round is no circle,
A square is not round,
Consequently, &c.

"Here the major proposition is converted by means of a *terminus infinitus* (*i.e.* contraposed), and its truth is manifested to us only through the consciousness that *all circles are round.* For, independently of this proposition, should we not hesitate,—there being innumerable things which are *not* round,—whether the circle were one of those which belonged to this category?

We think not; because we are aware. It is thus apparent that we use every syllogistic figure there, where the propositions, as each figure requires them, are more familiar and more current. The difference of the figures rests, therefore, not only on their form, but extends itself, by relation to their employment, also to things themselves, so that we use each figure where its use is more natural: *The First for finding out or proving the Attributes of a thing; the Second for finding out or proving the Difference of things; the Third for finding out and proving Examples and Exceptions; the Fourth for finding out and excluding Species of a Genus*[1]."

Mill has the following lines on Lambert and his work: "A German philosopher, Lambert, whose *Neues Organon* (published in the year 1764) contains among other things one of the most elaborate and complete expositions which have ever been made of the syllogistic doctrine, has expressly examined which sorts of arguments fall most naturally and suitably into each of the four figures; and his investigation is characterized by great ingenuity and clearness of thought. His conclusions are: 'The first figure is suited to the discovery or proof of the *properties* of a thing; the second to the discovery and proof of the *distinction* between things; the third to the discovery or proof of *instances* and *exceptions;* the fourth to the *discovery* or *exclusion* of the different species of a genus.' The reference of syllogism in the last three figures to the 'dictum de omni et nullo' is, in Lambert's opinion, strained and unnatural; to each of the three belongs, according to him, *a separate axiom, co-ordinate and of equal authority*, with that dictum, and to which he gives the names of 'dictum de diverso' for the 2nd figure, 'dictum de exemplo' for the 3rd, and 'dictum de reciproco' for the 4th. . . . . Mr Bailey (*Theory of Reasoning*, 2nd edition, pp. 70—74) takes a similar view of the subject[2]." A similar view is also taken by Archbishop Thomson and by Dr Martineau.

[1] Hamilton's *Lectures*, Vol. IV. p. 439.
[2] Mill's *Logic*, Vol. I. pp. 194—5.

§ 2. *Tho*[redacted]*ons.*—Thomson regards the following law as the general canon upon which all mediate inference depends:—"The agreement or disagreement of one conception with another is ascertained by a third conception, inasmuch as this wholly or by the same part agrees with both, or with only one, of the conceptions to be compared[1]."

For the first figure he modifies it thus:—"The agreement or disagreement of a subject and predicate is ascertained by a third conception, predicate to the former and subject to the latter; inasmuch as this wholly or by the same part agrees with both, or with one only, of the conceptions to be compared[2]."

For the second figure he modifies it thus:—"The agreement of two conceptions is ascertained by a third conception, which stands as predicate to both; inasmuch as this wholly or by the same part agrees with both, or with one only, of the conceptions to be compared[2]."

For the third figure he modifies it thus:—"The agreement of two conceptions is ascertained by a third conception, which stands as subject to both; inasmuch as this wholly or by the same part agrees with both, or with one only, of the conceptions to be compared[2]."

Thomson recognizes only three figures, and dismisses the fourth, on the ground that, in the conclusion in that figure, what was the predicate in a premiss becomes the subject, and what was the leading subject in a premiss becomes the predicate. This, he says, is not the natural order, but that order wholly inverted. The natural order is seen in the first, somewhat distorted in the second and third, and *wholly inverted* in the fourth, against which the mind rebels[3]. These special canons, as well as the general law, are quite self-evident, and do not require any explanation. They are directly applicable to the syllogism in each figure, and make Reduction unnecessary and superfluous.

1 Thomson's *Laws of Thought* (1864), p. 163.
2 *Ibid.* p. 175. 3 *Ibid.* pp. 177—8.

§ 3. *Whately's Canons.*—Whately regards the '*dictum de omni et nullo*' as the *ultimately* supreme Rule or Maxim of all reasoning; but as this is not *directly* applicable to all syllogisms, he gives the following two canons for all pure categorical syllogisms:—(1) "If two terms agree with one and the same third, they agree with each other; (2) if one term agrees and another disagrees with one and the same third, these two disagree with each other[1]." The first is for affirmative conclusions, and the second for negative. "On these two canons are built the syllogistic rules or cautions which are to be observed with respect to syllogisms, for the purpose of ascertaining whether those Canons have been strictly observed or not[2]." By these rules Whately determines the valid syllogisms in each figure, and then further confirms those in the 2nd, 3rd, and 4th figures by Reduction to the 1st, to which the 'dictum de omni et nullo' is directly applicable.

§ 4. *Hamilton's Canons.*—Hamilton divides all categorical syllogisms into Deductive and Inductive. The former are divided again into Intensive or Extensive according as the reasoning is in the quantity of comprehension or of extension. All *extensive* syllogisms are regulated by the canon "What belongs to the genus belongs to the species and individual; what is repugnant to the genus is repugnant to the species and individual, or more briefly, what pertains to the higher class pertains also to the lower[3]."

He then gives the following three proximate rules by which a *regularly and fully* expressed extensive categorical syllogism is governed:—(1) "It must have three and only three terms constituting three and only three propositions; (2) of the premisses, the sumption or major premiss must in quantity be definite, that is, universal, and the subsumption or minor premiss in quantity affirmative; (3) the conclusion must correspond in quantity with the subsumption, and in quality with the sumption[4]."

[1] Whately's *Elements*, 9th edn., p. 54.
[2] *Ibid.* p. 54.
[3] Hamilton's *Lectures*, Vol. III. p. 303.
[4] *Ibid.* p. 305.

According to Hamilton syllogisms in the first figure only are fully and regularly expressed, while all syllogisms in the 2nd, 3rd, and 4th figures are irregularly and imperfectly expressed. To the former the three rules are, therefore, directly applicable, while the latter must be regularly and fully expressed, or, in other words, reduced to the first figure, before the rules will be applicable to them. He, however, gives special rules for the 2nd, 3rd, and 4th figures. These rules are the same as those we have given in Part III. ch. III.

All *intensive* syllogisms are regulated by the canon "What belongs to the predicate belongs also to the subject; what is repugnant to the predicate is repugnant to the subject[1]."

In his later writings Hamilton adopts the doctrine of the quantification of the predicate, abolishes the fourth figure, divides the categorical syllogisms into (1) unfigured and (2) figured, and gives the following canons:—

I. "For the unfigured syllogism, or that in which the terms compared do not stand to each other in the reciprocal relation of subject and predicate, being in the same proposition, either both subjects or (possibly) both predicates, the canon is: In so far as two notions (notions proper, or individuals), either both agree, or one agreeing, the other does not, with a common third notion; in so far, these two notions do or do not agree with each other."

II. "For the figured syllogism, in which the terms compared are severally subject and predicate, consequently in reference to each other, containing and contained in the counter wholes of Intension and Extension, the canon is: What worse relation of subject and predicate subsists between either of two terms and a common third term, with which one, at least, is positively related, that relation subsists between the two terms themselves[2]."

Hamilton then gives a canon for each of the three figures. As examples of the unfigured syllogism he gives the following:—

[1] Hamilton's *Lectures*, p. 303.

[2] *Ibid.* Vol. IV. p. 357, and *Discussions*, pp. 653—5.

1. All C and some B are convertible;
   All B and all A are convertible;
   ∴ All C and some A are convertible.

2. A and B are equal,
   B and C are equal;
   ∴ A and C are equal.

§ 5. *Martineau's Canons*[1].—In the chapter on the Theory of Predication we have seen that Dr Martineau holds, for a certain class of propositions, the view, according to which the meaning of a proposition is that the attribute connoted by the predicate belongs to the substance or substances denoted by the subject. Consistently with this view, Dr Martineau would give the following axioms for the first three figures.

For the first figure, in its affirmative relations, the dictum would, according to him, appear in some such form as this:—"Where the same nature both has an attribute and is one, the attribute it has belongs to the substance in which it is." Thus in the mood *Barbara* the same nature B has an attribute A in the major premiss, and is itself one in the minor premiss, being its predicate; therefore the attribute A which B has, belongs to the substance C in which it (B) is, *i.e.* 'All C is A.'

For the second figure the dictum would be as follows:—"If the attribute be present with one nature and absent from another, neither of these can be the attribute of the other."

For the third figure it would be as follows:—"Where two attributes are co-present in the same sphere, each is an attribute of some thing having the other."

The true meaning, according to Dr Martineau, of the syllogism "All birds are warm-blooded, all swallows are birds, therefore all swallows are warm-blooded," is that in the major premiss the subject (birds) is wanted in its denotation, in the minor the same word 'birds' is wanted in its connotation, and in the conclusion the subject 'swallows' is wanted in its denotation and the predicate 'warm-blooded' in its connotation.

[1] *Essays*, Vol. II., 'Theory of Reasoning,' p. 352.

§ 6. *Mill's Canons.*—Mill gives the following two canons or fundamental principles of Syllogism or Ratiocination:—

(1) "A thing which co-exists with another thing, which other co-exists with a third thing, also co-exists with that third thing[1]."

(2) "A thing which co-exists with another thing, with which other a third thing does not co-exist, is not co-existent with that third thing[2]."

"The co-existence meant is," says Mill, "that of being jointly attributes of the same subject. The attribute of being born without teeth, and the attribute of having thirty teeth in mature age, are, in this sense, co-existent, both being attributes of man, though *ex vi termini* never of the same man at the same time[3]."

The first is the principle of affirmative syllogisms, and the second of negative syllogisms. Mill thus analyses an affirmative syllogism:—"All men are mortal, all kings are men; ∴ all kings are mortal. The minor premiss asserts that the attributes denoted by kingship only exist in conjunction with those signified by the word man. The major asserts that the last-mentioned attributes are never found without the attribute of mortality. The conclusion is, that wherever the attributes of kingship are found, that of mortality is found also[4]."

"If the major premiss," continues Mill, "were negative, as 'No men are omnipotent,' it would assert not that the attributes connoted by 'man' never exist *without*, but that they never exist *with* those connoted by 'omnipotent': from which, together with the minor premiss, it is concluded, that the same incompatibility exists between the attribute omnipotence and those constituting a king[4]." That is, the analysis of a negative syllogism, when fully stated, would be as follows:—No men are omnipotent, all kings are men; ∴ no kings are omnipotent. The minor premiss asserts that the attributes of kingship exist only in conjunction with those signified by 'man.' The major asserts that the last-

[1] Mill's *Logic*, Vol. I. p. 203. [2] *Ibid.* p. 204. [3] *Ibid.* p. 205.
[4] *Ibid.* p. 203.

named attributes never exist with those [redacted] by 'omnipotent.' The conclusion is, that the attributes of kingship never exist with those connoted by 'omnipotent,' or that wherever the former are found, the latter are not found.

For practical purposes, Mill gives the two canons quoted above in a different form founded upon the practical mode of expressing the meaning of a proposition. The real meaning of a proposition like 'All men are mortal' is that the attribute connoted by 'man' exists only in conjunction with the attribute connoted by 'mortal'; that wherever humanity is found, mortality is also found,—that is, the presence of the attribute 'humanity' is a sign or mark of the presence of the attribute 'mortality.' Hence the meaning of an affirmative proposition may, for practical purposes, be taken to be this, that 'the attribute connoted by the subject is a mark of the attribute connoted by the predicate'; and the meaning of a negative proposition, that 'the attribute connoted by the subject is a mark of the absence of the attribute connoted by the predicate.' For example, the proposition 'No men are perfect' means that the attribute 'humanity' is a mark of the absence of 'perfection.' In accordance with this mode of expressing the meaning of propositions, Mill gives the following two axioms or canons for practical purposes:—

(1) "Whatever has any mark has that which it is a mark of," when the minor premiss is a singular proposition with a proper name for its subject.

(2) "Whatever is a mark of any mark is a mark of that which this last is a mark of," when the minor premiss as well as the major is universal.

For example:—If the attribute A is a mark of the attribute B, and if an object has the attribute A, it has also the attribute B,—that is, an object that has the mark (A) has that (B) of which it (A) is a mark. Thus the meaning of the first syllogism, given above, would be as follows:—The objects 'kings' have the mark 'humanity,' which is a mark of 'mortality,' therefore the objects (kings) have the mark 'mortality;' or taking the term

'kings' also [redacted]otation, the attributes of a king which are a mark of humanity which is a mark of mortality are a mark of the last (mortality). The meaning of the second syllogism given above would be thus expressed:—The attributes of a king, which are a mark of the attributes of humanity, which are a mark of the absence of omnipotence, are a mark of the last (absence of omnipotence).

On this view the general formula of a syllogism is as follows:—

Attribute B is a mark of attribute A,
Attribute C is a mark of attribute B;
∴ Attribute C is a mark of attribute A.

Here B corresponds to the middle term, and A and C to the two extremes, the major and the minor terms. The first statement must be true *in all cases*, and the second *in all or in some cases*, and the conclusion accordingly in *all or in some cases.*

Barbara and *Darii* are thus expressed:

1. In all cases B is a mark of A,
   In all (or in some) cases C is a mark of B;
   ∴ In all (or in some) cases C is a mark of A.

Celarent and *Ferio*, thus:

2. In all cases B is a mark of the absence of A,
   In all cases (or in some cases) C is a mark of B;
   ∴ In all cases (or in some cases) C is a mark of the absence of A.

Mill gives canons for the first figure only, as the other figures can easily be reduced to that, and considers "the two elementary forms of the first figure as the universal types of all correct ratiocination,—the one when the conclusion to be proved is affirmative, and the other when it is negative, even though certain arguments may have a tendency to clothe themselves in the form of the 2nd, 3rd, and 4th figures; which, however, cannot possibly happen with the only class of arguments which are of first-rate importance, those in which the conclusion is an universal affirmative, such conclusions being susceptible of proof in the first figure alone."

B.—The Dilemma according to Logicians.

Whately[1] defines the true Dilemma as "a conditional syllogism with several antecedents in the major and a disjunctive minor."

Mansel[2] defines the Dilemma as "a syllogism, having a conditional major premiss, with more than one antecedent and a disjunctive minor."

Both Whately and Mansel give the following forms:—

I. Simple Constructive—

If A is B, C is D; and if E is F, C is D,
But either A is B, or E is F;
∴ C is D.

II. Complex Constructive—

If A is B, C is D; and if E is F, G is H,
But either A is B, or E is F;
∴ Either C is D, or G is H.

III. Destructive (always complex)—

If A is B, C is D; and if E is F, G is H,
But either C is not D, or G is not H;
∴ Either A is not B, or E is not F.

Whately *excludes* the following forms among others on the ground that they "hardly differ from simple conditional (that is Hypothetical-categorical) Syllogisms":—

(1) If A is B, C is D, E is F, and G is H,
But neither C is D, nor E is F, nor G is H,
∴ A is not B.

(2) If A is B, C is D,
If A is E, G is H,
But neither C is D, nor G is H,
∴ A is neither B nor E.

[1] *Elements*, p. 72. [2] Mansel's *Aldrich*, 1849, p. 93.

(3) If A is B, C is D and also E is F,
But either C is not D, or E is not F,
∴ A is not B.

"The Dilemma is sometimes exhibited," says Mansel, "in another form as a conditional syllogism in which the consequent of the major premiss is disjunctive, and *the whole* denied in the minor,—*e.g.* 'If A is B, either C is D, or E is F, or G is H; but neither C is D, nor E is F, nor G is H; therefore A is not B.' This form is given by Wallis[1] as well as by Wolf and Kant. But it is a perversion of the Dilemma proper, and introduces no distinction whatever, being merely a common disjunctive syllogism, as is shown by Wallis himself."

Professor Fowler[2] defines the Dilemma as "a complex syllogism of which one premiss is a conjunctive (hypothetical), and the other a disjunctive proposition." He follows in the main Mansel and Whately, differing from them only in one point, namely, that the antecedent of the conjunctive premiss may be single as well as double. Thus:—

If A is B, C is D and E is F,
But either C is not D, or E is not F;
∴ A is not B.

Here the antecedent is single.

Three other forms given by Professor Fowler are the same as those given by Mansel and Whately.

Professor Jevons follows Whately and Mansel, and adopts all their forms.

Thomson[3] defines the Dilemma as "a syllogism with a conditional premiss, in which either the antecedent or consequent is disjunctive." He gives the following forms of it:

(1) If A is B or E is F, then C is D,
But either A is B or E is F;
∴ C is D.

[1] Wallis's Lib., III. cap. 19.
[2] *Deductive Logic*, 6th ed., pp. 116—119.
[3] Thomson's *Laws of Thought*, pp. 203—5.

(2) If A is B, then C is D or E is F,
But neither C is D nor E is F;
∴ A is not B.

(3) If some A is B, either the M that are *A*, or the N that are *A*, are *B*,
But neither the M that are A, nor the N that are A, are B;
∴ A is not B.

Hamilton[1].—"If the sumption (*i.e.* the major premiss) of a syllogism be at once hypothetical and disjunctive, and if in the subsumption (minor premiss) the whole disjunction, as a consequent, be sublated (*i.e.* denied), in order to sublate the antecedent in the conclusion; such a reasoning is called an *Hypothetico-disjunctive syllogism*, or a *Dilemma*. The form of this syllogism is the following:—

"If A exist, then either B or C exists;
But neither B nor C exists;
∴ A does not exist."

"In the sifting of a proposed dilemma, the following points should be carefully examined:—(1) Whether a veritable consequence subsists between the antecedent and consequent of the sumption; (2) whether the opposition in the consequent is thorough-going and valid; and (3) whether in the subsumption the disjunctive members are legitimately sublated. For the example of a dilemma which violates these conditions, take the following:—

If virtue were a habit worth acquiring, it must insure either power, or wealth, or honour or pleasure;
But virtue insures none of these;
Therefore, virtue is not a habit worth attaining.

Here:—(1) The inference in general is invalid; for a thing may be worth acquiring though it does not secure any of those advantages enumerated. (2) The disjunction is incomplete; for there are other goods which virtue insures, though it may not

[1] Hamilton's *Lectures*, Vol. III. p. 350.

insure those [redacted]ed. (3) The subsumption is also vicious; for virtue has frequently obtained for its possessors the very advantages here denied."—*Hamilton*, Vol. III. pp. 352—3.

C.— Note on Mixed Syllogisms regarded by some Logicians as Immediate Inferences.

Hamilton in his later writings regards Mixed Syllogisms (the Hypothetical and Disjunctive Syllogisms, &c., of Logicians) as Immediate Inferences.

He says:—"It has been a matter of dispute among logicians, whether the class which I call *explicative* (*viz.* the Hypothetical and Disjunctive Syllogisms) be of Mediate or Immediate Inference. The immense majority hold them to be mediate; a small minority, of which I recollect only the names of Kant [Fisher, Weiss, Bouterwek, Herbart], hold them to be immediate. The dispute is solved by a distinction. Categorical inference is mediate, the medium of conclusion being a *term;* the Hypothetical and Disjunctive Syllogisms are mediate, the medium of conclusion being a *proposition*,—that which I call the *Explication*. So far they both agree in being mediate, but they differ in four points. The first, that the medium of the Comparative syllogism is a term; of the Explicative, a proposition. The second, that the medium of the Comparative is one; of the Explicative, more than one. The third, that in the Comparative the medium is always the same; in the Explicative, it varies according to the various conclusion. The fourth, that in the Comparative the medium never enters the conclusion; whereas, in the Explicative, the same proposition is reciprocally medium or conclusion[1]."

Again, (1) "They (Hypothetical and Disjunctive Syllogisms) are not composite by contrast to the regular syllogism, but more simple; (2) if inferences at all, they are *immediate* and not

[1] *Lectures*, Vol. iv. p. 378

mediate; (3) but they are not argumen~~tations, but~~ preparations (explications) for argumentation[1]."

Hamilton gives the following examples among others:—

"A.—Conjunctive Hypotheticals.

1. If A be D, A is C;	∴	A, being D, is C. A, not being C, is not D.
2. If B be A, B is not non-A;	∴	B, being A, is not non-A. B, being non-A, is not A.
3. If B be not A, B is non-A;	∴	B, not being A, is non-A. B, being non-A, is not A.
4. If E be not D, E is not C;	∴	E, not being D, is not C. E, being C, is D.

B.—Disjunctive Hypotheticals.

If B be either A or non-A;	∴	B, being A, is not non-A. B, being non-A, is not A.

'*If*' means *suppose that, in case that, on the supposition, hypothesis, under the condition, under the thought that, it being supposed possible;* ∴ &c., means *then, therefore, in that case,* &c. &c. *actually either*[2]."

Following Hamilton and others, Professor Bain treats of Hypothetical and Disjunctive Syllogisms under the head of Immediate Inferences, and exhibits them as follows:—

"In the Conditional Proposition—If A is B, C is D, the equivalent is, A being assumed to be B, it follows that C is D. There is no inference in this case. Accepting 'A is B,' we accept 'C is D'; this is another expression for the same fact." . . . "A second form of so-called conditional inference is, that the denial of the consequent is the denial of the antecedent; 'C is not D, therefore A is not B.' If the weather is fine, we go to the country; 'we are not going to the country, therefore, the weather is not fine.' This is still mere formal equivalence. It is implied in what has already been stated. It is not a distinct fact but the same fact, in obverse[3]."

[1] *Lectures*, Vol. IV. p. 383. [2] *Ibid.* pp. 390—91.
[3] Bain's *Deduction*, 2nd ed., p. 117.

"The Disjunctive Proposition may appear in the following forms:—

"I. A is either B or C.
II. Either B or C exists.
III. Either A is B, or C is D.

"'He is either a fool or a rogue,' means 'If not a fool, he is a rogue, and if not a rogue, he is a fool.' Otherwise, 'Not being a fool, he is a rogue,' and 'not being a rogue, he is a fool.' These are all equivalent forms; and the supposed reasoning consists merely in electing one alternative, according to the facts of the case. The datum being, 'he is not a fool,' we use the alternative 'he is a rogue,' and so on[1]."

"The Dilemma combines a conditional and a disjunctive proposition. If the *antecedent* of a conditional is made disjunctive, there emerges what Whately calls a simple *Constructive Dilemma*. If either A or B is, C is; now, either A or B is; therefore C is." "The *consequent* being made disjunctive, gives the more usual type:—If A is, either B or C is. If the barometer falls, there will be either wind or rain. Various suppositions may be made, bringing out the possible alternatives. Thus:—

(1) A is; then, B or C is.
(2) C is not; then, if A is, B is.
(3) C is; then, if A is, B is not.
(4) B is; then, if A is, C is not.
(5) B is not; then, if A is, C is.
(6) B is not and C is not; then, A is not.

"This last (6) is the *true dilemma* which is *Destructive*."

"Another form of simple dilemma is:—If B is, A is; and if C is, A is. Now, either B or C is. Whence A is[2]."

That Mixed Syllogisms are mediate inferences and not immediate, will be evident from the following considerations:—

I. In a mixed syllogism there are *three propositions*,—namely, the two premisses and the conclusion,—as in a pure syllogism. The conclusion does not follow from one premiss alone but from

[1] Bain's *Deduction*, 2nd ed., p. 119. [2] *Ibid.* p. 121.

the two taken together. In a hypothetical-categorical syllogism, for example, the major premiss is a hypothetical proposition, the minor premiss a categorical one, and the conclusion also a categorical: "If A is, B is; A is; therefore B is;" here the major premiss expresses the *dependence* of the existence of B on the existence of A, and is not a combination of two propositions as erroneously maintained by some logicians. The minor premiss 'A is' is a categorical proposition, affirming that A exists. It is not the same as the antecedent of the major premiss, which expresses the mere idea, thought, or simple apprehension of the existence of A. It is a proposition with a subject and a predicate, while the antecedent of the major premiss is merely a many-worded term. The two can not be regarded as identical, unless a term and a proposition are identical. The conclusion 'B is' is likewise not the same as the consequent of the major premiss. It is a categorical proposition affirming that B exists, while the consequent is a many-worded term, expressing the mere idea, thought, or simple apprehension of the existence of B.

The major premiss does not affirm that A exists or that B exists. Its antecedent and consequent are not two categorical propositions, but two many-worded terms. It expresses only the relation of dependence of the consequent on the antecedent, and says nothing as to the real existence of either. It lays down the general rule that wherever A is, B is,—that the existence of B accompanies every case of the existence of A. The minor premiss 'A is' asserts that *this* is a case of the existence of A. Whence it is inferred that there is a case of the existence of B, accompanying *this* case of the existence of A, or, in other words, that 'B is' (conclusion).

The minor premiss may be taken as a hypothetical proposition, with 'this case' understood for its antecedent; thus, "if this case is, A is." From this and the original hypothetical major premiss follows the conclusion, that "if this case is, B is," or, in other words, that 'B is' (conclusion), taken as a hypothetical proposition with 'this case' understood for its antecedent.

In the *destructive* form "If A is, B is; B is not; therefore A is not," the major premiss is hypothetical, and the minor premiss and the conclusion are categorical propositions as in the *constructive* form. The differences between the two forms are (1) that the minor premiss and the conclusion are affirmative in the constructive form, and negative in the destructive, and (2) that the minor premiss of the one and the conclusion of the other have the same subject and predicate, but *differ in quality*. Thus (1) the two affirmative propositions 'A is' and 'B is' are the minor premiss and the conclusion, respectively, in the constructive form, and the two negative propositions 'B is not' and 'A is not' are the minor premiss and the conclusion, respectively, in the destructive form. (2) 'A is' is the minor premiss in the constructive form, and 'A is not' is the conclusion in the destructive form; in the former 'B is' is the conclusion, and in the latter 'B is not' is the minor premiss. The conclusion of the one has the same subject and predicate as the minor premiss of the other. From this fact has probably arisen the mistaken notion that in these syllogisms 'the minor premiss and the conclusion indifferently change places[1].' Hamilton says: "The fourth, that in the Comparative the medium never enters the conclusion; whereas in the Explicative (*i.e.* hypothetical syllogisms, &c.) the same proposition is reciprocally medium or conclusion." Now, the proposition is not the same. Its subject and predicate only are the same, but its quality is different. The minor premiss of the one, and the conclusion of the other, can not be regarded as the same, unless an affirmative and a negative proposition, having the same subject and predicate, are the same,—unless A and E, A and O, E and I, I and O, are identical. With equal justice might the conclusion in one, and the minor premiss in the other, of the two forms, namely, *affir-*

[1] This point is differently interpreted by Professor Robertson (*Mind* for 1877, p. 264) and Mr Keynes (*Formal Logic*, p. 234). They consider it to be a blunder, from which, I think, Hamilton is free, as is evident from the examples given by him and quoted in this book on page 289.

mative and *negative*, of the following categorical syllogisms, be regarded as identical :—

1. *Affirmative* Categoricals :—

(1) All men are mortal,
All kings are men,
∴ All kings are mortal.

(2) All men are mortal,
Some kings are men,
∴ Some kings are mortal.

2. *Negative* Categoricals :—

(1) All men are mortal,
All kings are not mortal,
∴ All kings are not men.

(2) All men are mortal,
No kings are mortal,
∴ No kings are men.

1. Corresponding *Constructive* Hypothetical-categoricals :—

(1) If all kings are men, all kings are mortal;
All kings are men;
∴ All kings are mortal.

(2) If some kings are men, some kings are mortal;
Some kings are men;
∴ Some kings are mortal.

2. Corresponding *Destructive* Hypothetical-categoricals :—

(1) If all kings are men, all kings are mortal;
All kings are not mortal,
∴ All kings are not men.

(2) If some kings are men, some kings are mortal;
No kings are mortal,
∴ No kings are men.

The minor premiss in one and the conclusion in the other of the *affirmative* and *negative* categoricals have the same subject and predicate, and stand to each other in the same relation in which the minor premiss in one and the conclusion in the other of the *constructive* and *destructive* hypothetical-categoricals stand to each other. But who would maintain that in those categorical syllogisms, "the minor and the conclusion indifferently change places," or that "the same proposition is reciprocally medium or conclusion"?

II. In a mixed syllogism there are *three terms* as in a pure syllogism. In the example taken above, the consequent as a many-worded term, is the major term, the antecedent as a many-worded term, is the middle term, and 'this case', or 'the case in

question' un[illegible]is the minor term. This will be evident, if the mixed syllogism is reduced to the *pure* form:—

(i) Categorical:

Every case of the existence of A is a case of the existence of B; the case in question (or this case) is a case of the existence of A: therefore the case in question (or this case) is a case of the existance of B.

Here the three terms are—(1) case of the existence of B (major term), (2) case of the existence of A (middle term), and (3) the case in question or this case (minor term). (2) is the middle term to which (1) and (3), the two extremes, are related;—that is, a relation between (1) and (3) is established from a relation of each of them to a third (2) or middle term, as in the case of a categorical syllogism.

(ii) Hypothetical:

If A is, B is; if this case is, A is: therefore if this case is, B is.

This is a pure hypothetical syllogism in *Barbara*. Here the middle term is the antecedent in the major premiss, and consequent in the minor, as it should be in that mood.

From this it is evident, that the objection that a mixed syllogism has no middle term, and consists of two terms only, is entirely unfounded. It has arisen from a misunderstanding of the true nature of the hypothetical major premiss, which has been erroneously regarded as consisting of two propositions instead of two many-worded terms. It is also evident that the middle term is not, as Hamilton says, a proposition, but a many-worded term.

III. If A is B, C is D;
∴ A being B, C is D.

This is the form in which a mixed syllogism regarded as an immediate inference is stated; and it is argued that the conclusion follows immediately from the premiss, and that no minor premiss is necessary. Now, it can be shown that a categorical syllogism may likewise be stated in the above form;

and should it, therefore, be regarded as an immediate inference?

All men are mortal,
∴ All kings, being men, are mortal.

Here also the conclusion follows from the premiss. But it is evident that the conclusion is but a short or abridged statement of two propositions, namely, the minor premiss, 'all kings are men,' and the conclusion, 'all kings are mortal.' Some logicians indeed actually maintained that even in the categorical syllogism, the minor premiss is unnecessary, that the conclusion follows from the major premiss. Thus they would regard categorical syllogisms as consisting of two propositions only, and consequently as immediate and not as mediate inferences. But we have seen (pp. 258—9) that the conclusion does not follow from the major premiss alone, nor from the minor alone, but from the major and the minor taken jointly. And this is true of mixed syllogisms as well as of categoricals. The conclusion 'A being B, C is D,' is merely a short or abridged statement of two propositions, namely, the minor premiss 'A is B,' and the conclusion 'C is D.'

Here may be noticed an objection raised by Professor Bain. He sees no *real* inference in mixed syllogisms. By real inference he means a proposition that is not contained in, or implied by, the premiss or premisses. This objection is founded on a misunderstanding of the true nature of deductive inference. It is equally applicable to categorical syllogisms. In these also the conclusion is not a *real* inference, but a proposition which is contained in, or implied by, the two premisses. Without disputing about words, it may be said that the inference is *mediate* and *real* in mixed syllogisms, if it is mediate and real in categoricals.

D.— NOTE ON THE REDUCTION OF INDUCTIVE REASONING TO THE SYLLOGISTIC FORM.

The fundamental principles of Inductive Reasoning (whatever be their origin and nature) are the two Laws of Causation and Uniformity of Nature. The first law includes the two propositions—(1) every phenomenon has a cause, and (2) the cause of a phenomenon is the invariable, or, as Mill says, the unconditionally invariable antecedent of the phenomenon. The second law means that (3) the same cause or antecedent will, under the same circumstances, produce the same effect. All inductive reasonings are conducted either directly in accordance with one or other of these laws or with laws that follow from them. For example, from the second proposition of the first law follow such laws as the following given by Professor Bain[1]: (4) 'whatever antecedent *can be left out*, without prejudice to the effect, can be no part of the cause;' (5) 'when an antecedent *can not be left out* without the consequent disappearing, such antecedent must be the cause or a part of the cause;' (6) 'an antecedent and a consequent rising and falling together in *numerical concomitance* are to be held as cause and effect,' and also the following: (7) 'if two or more instances of a phenomenon under investigation have only one circumstance in common, that circumstance is the cause (or effect) of the phenomenon;' (8) 'if an instance where a phenomenon occurs, and an instance where it does not occur, have every circumstance in common except one, that one occurring only in the first; the circumstance present in the first and absent in the second, is the cause, or a part of the cause, of the given phenomenon'[2].

[1] Bain's *Induction*, 2nd ed., pp. 47, 48, 57.

[2] That the propositions marked (4), (5), (6), (7), and (8) follow from the proposition marked (2) can be shown as follows:—

(4) is the converse of the obverse of (2). Obvert (2), and then convert the obverse;—the cause of a phenomenon is not the variable antecedent of the phenomenon—[E, obverse of (2)]. (4) That which

Examples of Inductive Reasoning:-

(1) The antecedents		A B C	produce the consequents		a b c
"	"	A B D	"	"	a b d
"	"	A D E	"	"	a d e
"	"	A E F	"	"	a e f

∴ The antecedent A is the cause of the phenomenon a according to the principle—a derivative one—marked (7) above, and called the Canon of the Method of Agreement. This inductive reasoning may be easily reduced to a syllogism which has for its major premiss the canon, and for its minor the data of the reasoning, that is, the instances of the phenomenon. The syllogism is a hypothetical-categorical one, and is as follows :—

If two or more instances of a phenomenon under investigation have only one circumstance in common, that circumstance is the cause of the phenomenon (major premiss).

The four instances given of the phenomenon a under investigation have only one circumstance, namely, A, in common (minor premiss).

is the variable antecedent of a phenomenon, or, in other words, which 'can be left out without prejudice to the effect,' is not the cause of the phenomenon (E, converse of the obverse).

(5) is the converse of (2), which, being a *definition*, may be converted *simply*. (5) That which is the invariable antecedent of a phenomenon, or, in other words, which 'can not be left out without the consequent disappearing,' is the cause of the phenomenon [A, converse of (2)].

(6) is a mathematical inference from (2). The cause and the effect increase or decrease together. $A=B$ ∴ $2A=2B$, or $nA=nB$.

(7) follows from (4) and (5) taken together. By (4) the circumstances which are not common to all the instances of the phenomenon, that is, which 'can be left out without prejudice to the effect,' can be no part of the cause. By (5) the circumstance which is common to all the instances, that is, which 'cannot be left out without the consequent disappearing,' is the cause or a part of the cause of the phenomenon.

(8) follows likewise from (4) and (5) taken together.

∴ That circumstance A is the cause of the phenomenon a (the conclusion).

Or, the syllogism may be stated in the form of a categorical as follows :—

The invariable antecedent of a phenomenon is the cause of the phenomenon (major premiss).

A is the invariable antecedent of the phenomenon a (minor premiss).

∴ A is the cause of the phenomenon a (the conclusion).

(2) The antecedents A B C produce a b c
" " B C " b c,

∴ The antecedent A is the cause or a part of the cause of the phenomenon a according to the principle—also a derivative one—marked (8) above, and called the Canon of the Method of Difference. This inductive reasoning may be likewise reduced to the syllogistic form as follows :—

If an instance where a phenomenon occurs, and an instance where it does not occur, have every circumstance in common except one, that one occurring only in the first; the circumstance present in the first and absent in the second is the cause, or a part of the cause, of the given phenomenon (major premiss).

An instance A B C......a b c, where the phenomenon a occurs, and an instance B C......b c, where it does not occur, have every circumstance in common except one, namely, A, that one occurring only in the first (minor premiss).

Therefore, the circumstance A present in the first and absent in the second is the cause, or a part of the cause, of the given phenomenon a (conclusion).

Or, as follows :—

When an antecedent can not be left out without the consequent disappearing, such antecedent must be the cause, or a part of the cause, of the consequent (major premiss).

The antecedent A can not be left out without the consequent a disappearing (minor premiss).

Therefore the antecedent A must be [illegible] or a part of the cause, of the consequent a.

Similarly, other inductive reasonings may be reduced to the syllogistic form.

Let us take as a concrete example the first one we have given in the chapter on the Different Kinds of Reasoning (page 123):—

Air expands by heat,
Water expands by heat,
Mercury expands by heat,
Copper expands by heat,
&c. &c.

∴ All material bodies expand by heat[1].

Here the antecedent circumstances are the material bodies *plus* heat, and the consequents or effects are the same bodies *plus* the phenomenon of expansion. All the antecedents agree in the circumstance of being heated material bodies; and, therefore, according to the Canon of the Method of Agreement, this circumstance is the cause of the phenomenon of expansion, that is, in the given instances, heat being the invariable antecedent of expansion is the cause of this phenomenon. More accurately, the different steps of the argument may be stated as follows:— (1) Air and other bodies expand by heat, the expansion of these bodies is a phenomenon; therefore it has a cause, according to the principle 'every phenomenon has a cause;' (2) the invariable antecedent of this phenomenon is the application of heat, as shown by the given instances; therefore, according to the principle, namely, 'the invariable antecedent of a phenomenon is the cause of the phenomenon,' the application of heat to material bodies is the cause of the expansion in the given instances; and (3) according to the principle, namely, 'the same antecedent or cause will, under the same circumstances, produce the same effect,' it may be inferred that the application of heat to other

[1] This proposition is not universally true. See an exception on page 75. But that does not affect the line of reasoning adopted here.

material bodies, as well as to the same in future, will produce expansion; or, in other words, all material bodies expand by heat. The different steps may be thus stated syllogistically:—

(1) Every phenomenon has a cause, the expansion of air and other bodies by heat is a phenomenon; therefore it has a cause.

(2) The invariable antecedent of a phenomenon is the cause of the phenomenon, the application of heat is the invariable antecedent of the phenomenon of expansion in the given instances; therefore the application of heat is the cause of the phenomenon of expansion in the given instances.

(3) The same antecedent or cause will, under the same circumstances, produce the same effect or consequent,—that is, if a certain antecedent produces, under certain circumstances, a certain consequent, then it will, under the same circumstances, produce the same consequent; the antecedent, namely, the application of heat to material bodies, under the circumstances of there being no counteracting agencies, produces the consequent, namely, the expansion of those bodies; therefore the same antecedent, namely, the application of heat to material bodies, under the same circumstances of there being no counteracting agencies, will produce the same consequent, namely, the expansion of those material bodies.

Thus all inductive reasonings, like mathematical (see p. 123), may be reduced to the syllogistic form: usually their conformity to an axiom, principle, law, canon, or rule recognized as true is regarded as a sufficient proof of their validity, even as constituting their validity itself; but in all cases where they are valid, they are capable of being reduced to the syllogistic form. In Physics, for example, conformity to the principles of causation and of uniformity of nature, or to the canons and rules derived from them, is regarded as constituting the validity of the reasonings; but we have seen that, taking the principles or the canons as major premisses, and the data as minor, we can, in all cases, construct syllogisms which have the same conclusions as the reasonings themselves; and the best test of the validity of the reasonings is the possibility of their reduction to the syllogistic

form: any weakness in the argument is sure to be brought to light by this process.

To see clearly what premisses have been assumed, or, on what data—both principles and facts—the conclusion ultimately rests, it is necessary to reduce a reasoning or a train of reasoning to the syllogistic form. In this form every step of the argument will be clearly exhibited and every proposition required to prove the conclusion laid bare, and should there be any error in the *process* of reasoning, it will be brought to light by the axioms, canons, or rules of Deductive or Syllogistic Logic. Of course, if there be any falsity or fallacy in the ultimate data—if any universal principle or any particular fact has been unwarrantedly assumed—it can not be detected by those axioms, canons or rules; nor can it be detected by the canons and rules of any Logic, as understood by British Logicians. For the *particular fact*, the ultimate appeal must be made to observation, external or internal; and for the *universal principle* the appeal is made (1) to the Experience of the Individual, that is, to Repeated Experience and Generalisation (the Empirical or Experiential Theory); or (2) to Intuition, that is, to Immediate Knowledge by the Reason (the Intuitional Theory); or (3) to the Forms and Categories of the Mind (the A-priori or Kantian Theory); or (4) to the Experience of the Race, that is, to Inherited Tendencies and Experience (the Evolutional Theory). The first question can be decided only by the special science to which the fact belongs; and the second question by the science which treats of the origin and nature of universal principles, and which has been variously called Metaphysics, the Science of First Principles, the Science of the most General Laws, &c.

E.—The Nature and Province of Objective Logic.

The *name* 'Objective Logic,' and the *thing* signified by it, are comparatively new. I intend, therefore, to give here extracts from the writings of Logicians with a view to indicate the nature and province of the *thing* as conceived by them.

§ 1. *Hamilton's View.*

"The doctrine...which expounds the laws by which our scientific procedure should be governed, in so far as these lie in the forms of thought, or in the conditions of the mind itself, which is the subject in which knowledge inheres,—this Science may be called *Formal*, or *Subjective*, or *Abstract*, or *Pure* Logic. The Science, again, which expounds the laws by which our scientific procedure should be governed, in so far as these lie in the contents, materials, or objects, about which Knowledge is conversant,—this Science may be called *Material*, or *Objective*, or *Concrete*, or *Applied* Logic[1]."

§ 2. *Mill's View.*

In Mill's writings the *name* 'objective Logic' rarely, if ever, occurs; but the *thing* is to be found in abundance. He defines and treats of the *thing* in his *Examination of Hamilton's Philosophy* and also in his *System of Logic*, and expounds and criticises logical doctrines from that point of view. There is, however, a difference between the *thing* as conceived in the *Examination*, and the *thing* as treated of in the *Logic*. In the former he speaks of concepts, judgments, and reasonings, and requires that they should be right or true, that is, that they should *agree* with fact or reality. In the latter he treats of phenomena or facts themselves: names, for instance, stand for things; propositions for relations of things; and arguments are about the relations of those relations. In the *Logic* he gives up

[1] Hamilton's *Lectures*, Vol. IV. p. 231.

concepts and judgments, and condemns the theories of predication, which are founded upon ideas of things, and not upon things or phenomena themselves. The *Logic*, therefore, treats of things and their relations; and it is from this point of view that he finds the Syllogism guilty of the *petitio principii*, and Immediate Inference as no inference at all.

Mill's conception of Logic has thus two phases:—

(1) In the first phase Logic is conceived to treat of concepts, judgments, and reasonings as *agreeing* with things.

(2) In the second phase, Logic is conceived to treat of things or phenomena themselves, and of their relations and correlations.

Among English Logicians Mill, in fact, seems to occupy an intermediate position between such Subjective Logicians as Hamilton and Mansel, and such Objective Logicians as Spencer and Lewes[1].

§ 3. *Spencer's View.*

"A distinction exists which, in consequence of its highly abstract nature, is not easily perceived, between the science of Logic and an account of the process of Reasoning......The distinction is, in brief, this, that Logic formulates the most general laws of correlation among existences considered as objective; while an account of the process of Reasoning, formulates the most general laws of correlation among the ideas corresponding to those existences. The one contemplates in its propositions, certain connexions predicated, which are necessarily involved with certain other connexions given; regarding all these connexions as existing in the *nonego*—not, it may be, under the form in which we

[1] On the difference between Formal Logic (Hamilton's view) and Material Logic (the first phase of Mill's view of Logic), see Venn, *Logic of Chance*, 2nd ed. chapter x., "Discussion of some of the Principal Views as to the Nature and Province of Logic, Material and Conceptualist." On the difference between the two phases, briefly indicated above, of Mill's conception of Logic, compare Ueberweg's distinction of Logic and Metaphysics. See *Logic*, §§ 1, 2, 3, 8.

know them, but in some form. The other contemplates the process in the *ego* by which these necessities of connexion come to be recognised.

"Why this distinction has eluded observation, it is not difficult to see. Logic on the one hand, and the theory of Reasoning on the other, deal with relations from which all concrete terms are, as far as possible, expelled. They are severally obliged to use some terms (which, however, are by preference symbolic, so that they may express indifferently any kind of existence, attribute, action, or even relation); otherwise the relations dealt with can not be expressed, or distinguished from one another. But they intentionally ignore the natures of the terms, and occupy themselves with the most general dependencies of these most abstract relations. The result is that, in the absence of terms definitely specified as belonging either to the outer world or to the inner world, the two sets of relations, belonging the one to the outer world and the other to the inner world, become indistinguishable. Hence there arises this confusion between Logic, which is as much a division of the science of objective existence as Mathematics, and the theory of Reasoning, which is a division of subjective Science.

"To show that the affirmations of Logic refer to the connexions among things considered as existing apart from our consciousness, and not to the correlative connexions among our correlative states of consciousness, we need but to take the case of logical propositions as numerically quantified, in the system of Prof. de Morgan. I quote Mr Mill's condensed statement of the doctrine; for Prof. de Morgan's own statements are so encumbered with details and symbols, that I can not find in his work one that is at once brief and adequate.

"'From the premises most B's are C's, most B's are A's, it may be concluded with certainty that some A's are C's, since two portions of the class B, each of them comprising more than half, must necessarily in part consist of the same individuals. Following out this line of thought, it is equally evident that if we knew exactly what proportion the 'most' in each of the premises

bear to the entire class B, we could increase in corresponding degree the definiteness of the conclusion. Thus if 60 per cent. of B are included in C, and 70 per cent. in A, 30 per cent. at least must be common to both; in other words, the number of A's which are B's, and of C's which are A's must be at least equal to 30 per cent. of the Class B.'[1]"

"......But the clearest proof that relations among objective existences form the subject-matter of Logic, is yielded by the mechanical performance of logical inference. Prof. Jevons has devised a machine of such kind that, its keys being pressed down in proper order in conformity with the premisses of the given logical proposition, the conclusion is presented by the combinations which the machine displays. Here it is undeniable that the relation disclosed is an objective one; and it is equally undeniable that the thing ascertained is, that this objective relation was necessarily involved in those other objective relations which constitute the premisses. We have nothing to do with thought at all. We have to do with inter-dependencies among outer things or agencies. The machine having been set to represent objects and attributes in certain relations, evolves certain necessarily-accompanying relations, such as would otherwise be ascertained by actual examination of the objects and attributes[2]."

"The propositions of Logic, then, primarily express necessary dependencies of things, and not necessary dependencies of thoughts; and in so far as they express necessary dependencies of thoughts, they do this secondarily—they do it in so far as the dependencies of thoughts have been moulded into correspondence with the dependencies of things. I say advisedly, '*in so far as*'; for there are certain absolute unlikeness of nature between the outer dependencies and the inner dependencies which for ever forbid anything more than a symbolic correspondence, as we shall hereafter see more clearly. The greater part

[1] *Principles of Psychology*, 2nd ed. Vol. II. § 302, pp. 87—88.

[2] *Principles of Psychology*, Vol. II. § 302, p. 90.

of the necessary objective correlations are *statical*, while all the necessary subjective correlations are dynamical; and only in so far as dynamical correlations may be so arranged as to symbolize statical correlations, can the necessary dependencies of Reason be made to parallel the necessary dependencies of Logic[1]".

"......See, then, the inevitable implication. No one questions the fact that while I was using these marbles to exemplify arithmetical truths and geometrical truths, I was contemplating, and was teaching, necessary objective correlations. Can it be that when I used these same marbles to exemplify necessities of correlation among groups and sub-groups, distinguished by certain marks, I passed from the region of objective necessities to the region of subjective necessities? No one will, I think, have the hardihood to assert as much. There is no choice but to leave these most general laws of correlation which Logic formulates, outside along with the laws of numerical correlation and geometrical correlation; or else, bringing them into the mind as laws of thought, to bring with them these mathematical laws as laws of thought in the same sense, and, by other steps equally unavoidable, to merge all objective facts in subjective facts: thus abolishing the distinction between subject and object[2]".

Note. Mr Carveth Read adopts Spencer's view of Logic, with these two qualifications, *first*, that Logic "may very well consider the correlation of ideas among themselves," and *second*, that Logic "deals only with laws of phenomena." See *Mind*, Vol. II. "On some Principles of Logic," p. 336. For a Critical Notice of Mr Read's "*Theory of Logic: an Essay*," by Dr Venn, see *Mind*, Vol. III. p. 539. See also a note on "'Matter-of-fact' Logic," by Mr J. N. Keynes, in *Mind*, Vol. IV. p. 120. For a criticism of Spencer's view of Logic, by Dr Venn, see *Mind*, Vol. IV., "The Difficulties of Material Logic," p. 35. Dr Venn suggests a view of Logic which seems to correspond to Ueberweg's view and to the first phase of Mill's conception of Logic (see Mill's View). "Instead of regarding Logic as a purely objective

[1] *Principles of Psychology*, Vol. II. § 302, pp. 90—91.

[2] *Principles of Psychology*, Vol. II. § 302, pp. 92, 93.

science," says Dr Venn, "we might with more propriety term it a science which gives the rules for converting the subjective into the objective" (*Mind*, Vol. IV. p. 46). Compare Ueberweg's definition, namely, "Logic is the science of the regulative laws of human knowledge" (*Logic* § 1), and Mill's view of Logic as "the science of the conditions on which right concepts, judgments, and reasonings depend" (*Examination of Hamilton's Philosophy*, 4th ed. p. 464).

§ 4. *Lewes's View.*

"Let us pause for a moment to consider the very different meanings assigned to the word Logic. It commonly stands for:

(1) the art of reasoning;

(2) the theory of reasoning;

(3) Reasoning itself;

(4) the laws of mental operation, irrespective of the symbols operated on (Formal Logic);

(5) the rules of Proof.

"The first of these I hold to be absurd. There is no more an art of Reasoning than there is an art of Breathing, or Digesting. But so little is this understood that even thoughtful writers will be found declaring that we must learn how to reason, as we learn how to fence or to swim. In consequence of this misconception, certain studies, notably Mathematics, are popularly believed 'to strengthen the Faculty,' to develop the logical powers, to 'invigorate the judgment.' The psychological notions which lie at the basis of such declarations are sadly defective.

"The second and third meanings of the word are objectionable because restricting Logic to the process of Ratiocination when the ratios are abstract. This restriction is got rid of in the fourth and fifth meanings, which may be accepted as comprehensive. The fourth designates the universal Logic, it includes all Laws of Grouping (λέγειν means to bind together, to group), and is therefore applicable to Feeling and Thought (in the subjective world), and to Cause (in the objective world).

"The [redacted] technical and restricted meaning of a *Codification of the rules of Proof.* In this last sense only can Logic be a separate Discipline. It may be likened to the science of Grammar apart from Language. Thus the speech of men of various nations embodies and exhibits certain general rules; or tendencies, according to which words are grouped. These tendencies grammarians detach and treat separately as Laws of Speech, Rules of Grammar. Logicians may in like manner detach certain general procedures of the investigating intellect, and treat them apart as the Rules of Rational Inquiry.

"Having fixed on the meaning Logic may bear when employed for a Special Discipline, namely, the codification of the rules of Proof, we may complete it by assigning to Metaphysics the parallel position of a codification of the laws of Cause. It will thus occupy very much the place assigned to it by Hegel, namely, that of Objective Logic. The Object and the Subject would have one general Logic, separately viewed as the Logic of Intelligence, and the Logic of the Cosmos. In the Cosmos, viewed objectively, things influence each other and events succeed each other according to invariant tendencies, or laws. When these phenomena are reproduced in consciousness they are also reproduced according to invariant tendencies; and thus it is that a law of Cause becomes a rule of Proof. Logic in its widest sense is Grouping. The laws of Grouping are the general tendencies of Things and the general tendencies of Thought. The common separation of Thought from the things thought of, is an artifice; but it is one so deeply inwoven with our philosophy and practice, that the mind untutored in such researches, is astonished and distressed at the statement of the identity between Thing and Thought, Object and Subject. With what qualifications this statement has to be received we shall hereafter discuss. Here I am only concerned to define the position of Metaphysics as Objective Logic—the codification of the most abstract laws of Cause. The Subjective Logic takes no account of the special instruments and processes by which each science reaches Proof, it is occupied solely with the codification of the processes. In

like manner the Objective Logic disregards special details in the processes of Causation, solely occupied with codifying the most abstract results. Subjective Logic rejects whatever lies beyond the range of verification, and thus demarcates Reality from Possibility, Fact from Fiction. Objective Logic rejects whatever lies beyond that world of sensibles and extra-sensibles which can come within the range of Experience; and thus demarcates Metaphysics from Metempirics.

"This distinction between the two aspects of Logic represents the distinction between Knowing and Being; and the identity underlying this diversity is also represented. In one we find the laws of Investigation; the abstract conditions to which all knowledge is subject. In the other we find the laws of the Investigated, the abstract conditions to which the knowledge is subject. Only on the assumption of the invariability of relations objective and subjective is Philosophy possible. In the most abstract of the sciences, that of Number, this identity is manifest. No arithmetical operation would be valid were there not this accord between the internal and the external; and the assumption of such an accord runs throughout Science. Indeed the axioms of Logic and the axioms of Science are the concave and convex aspects of the same curve[1]."

In a footnote to the above, Lewes remarks:—"Since this view was written Mr Spencer has propounded a new view of Logic. Starting from the proposition that the Syllogism refers to the dependencies of Things and not of Thoughts, he comes to the conclusion that Logic must be carried over entirely to the Objective world. He therefore places it beside Mathematics—as it is placed in Comte's latest scheme. He holds that 'it formulates the most general laws of correlation among existences considered as objective.' Referring the reader to Mr Spencer's exposition (*Psychology*, II. §§ 302 *et seq.*), I will merely here add that my chief divergence from it arises from my inability to

[1] Lewes's *Problems of Life and Mind*, 3rd ed. Vol. I. pp. 72—75.

accept his [redacted] there being only a symbolic correspondence between the inner and outer worlds. I hope to make it clear that the correspondence is real[1]."

§ 5. *Summary.*

According to Hamilton, Objective Logic is the science of the forms of the objects known, and Subjective Logic the science of the forms of the Knowing subject. According to Spencer, Logic is the science of "the most general laws of correlation among existences considered as objective," and the Theory of Reasoning the science of "the most general laws of correlation among the ideas corresponding to these existences." Spencer's Logic and Theory of Reasoning seem to correspond to Hamilton's Objective Logic and Subjective Logic, respectively. According to Spencer, Logic, like Mathematics, is an objective science, and treats of the most general laws of objects existing in the outer world. It is as little dependent upon mental processes as Mathematics. Its processes and laws are determined by the processes and laws of objects and not of thoughts.

Lewes regards Objective Logic as identical with Metaphysics. "The Object and the Subject would have one general Logic, separately viewed as the Logic of Intelligence and the Logic of the Cosmos." This general Logic is Objective Logic applicable alike to the Subject and to the Object, to both thoughts and things. Subjective Logic is concerned, according to him, with the codification of the rules of Proof, of the processes of Knowing, and Objective Logic with the codification of the most abstract laws of Cause, of the processes of Being. This distinction between Subjective and Objective Logic seems to correspond to Hamilton's and Spencer's distinction of these two Logics.

According to Lewes, Thought and Things, Knowledge and Being are, like the concave and convex aspects of the same curve, the subjective and objective aspects of the same existence; and the Logic of the one really corresponds to, or is identical

[1] *Problems of Life and Mind*, 3rd ed. Vol. I. p. 75.

with, the Logic of the other. While, a[redacted]Spencer, the Subject and the Object, the Ego and the Non-ego are two separate realities; and the Logic of the one has only a certain symbolic correspondence or parallelism to the Logic of the other.

F.—Text, p. 104

There are two classes of verbal propositions:—(1) those that explain the meanings of names, which *may* or *may not* agree with facts, and (2) those that explain the meanings of names, which *do* agree with facts. In the text, I have in view the second class of verbal propositions.

G.—Note on Obversion and Contraposition.

In Obversion the contradictory of the predicate of the premiss is made the predicate of the obverse. In Contraposition the contradictory of the predicate of the premiss is made the subject of the contrapositive. Both Obversion and Contraposition therefore imply that the predicate of the premiss has a contradictory term—that it does not cover the whole sphere of thought and existence. And, as the subject and the predicate of the premiss may, by conversion, become the predicate and the subject, respectively, of the converse, and as this converse may be obverted or contraposed, it is clear that Obversion and Contraposition also imply that the subject of the premiss has a contradictory term. These two forms of Immediate inference are, therefore, founded on the assumption that every term, whether subject or predicate of a proposition, has a contradictory. If the term B in the proposition "All A is B" has no contradictory, that is, if it covers the whole sphere of thought and existence, then the proposition can be neither obverted nor contraposed. For the same reason, the proposition "Some A is

B" can not [redacted] A and I can not, therefore, be obverted and A can not be contraposed, unless every term has a contradictory.

Has every term a contradictory? May not the whole sphere of thought and existence be covered by one term? Are we justified in assuming that the term B in the proposition "All A is B" has a contradictory? Such an assumption seems to lead to a conclusion which violates one of the fundamental rules of Deductive Inference, in the following train of immediate inferences:—

The proposition "All A is B" being given as true, the following propositions are true—

(1) "No A is not-B," the obverse of the premiss (or the given proposition).

(2) "No not-B is A," the converse of (1) or the contrapositive of the premiss.

(3) "All not-B is not-A," the obverse of (2).

(4) "Some not-A is not-B," the converse of (3).

(5) "Some not-A is not B," the obverse of (4).

In the last conclusion, the term B is distributed, while it is undistributed in the original premiss. This can not be allowed in any form of Deductive Inference, mediate or immediate. If a term is not distributed in the premiss, it can not be distributed in the conclusion; that is, if a term is taken in the premiss to mean *at least one* thing denoted by it, it can not in the conclusion be taken to mean *all* things denoted by it. The conclusion marked (5) is, therefore, inadmissible. It is obtained from the original premiss by the processes of obversion and conversion; and the fallacy lies not in the process of conversion but in that of obversion, which assumes that the term B has a contradictory and is therefore limited in its sphere, although in the original premiss its limitation is not implied and it *may cover* the whole sphere of thought and existence.

H.—Text, p. 224.

Mr Keynes gives two examples of Sorites in which *all* the constituent syllogisms are in the 2nd and the 3rd figure respectively. See his *Formal Logic*, pp. 219—220. It is worth noting that, by merely transposing the premisses, his examples can be reduced to the forms given above. His first example is:—"All A is B, no C is B, all D is C, all E is D, all F is E, therefore, no A is F." Write it as follows:—"All F is E, all E is D, all D is C, no C is B, all A is B, therefore no A is F." In this only the last syllogism is in the 2nd figure, the others are in the first. His second example is:—"All B is A, all B is C, all C is D, all D is E, therefore, some E is A." Write it as follows:—"All B is C, all C is D, all D is E, all B is A, therefore, some E is A." In this also only the last syllogism is in the 3rd figure, the others in the 1st.

INDEX.

Absolute terms, 27, 36, 42
Abstract terms, 27, 30, 38, 39, 41
———, singular and general, 32
Accidens, separable and inseparable, 83
Accident, fallacy of, 233
Accidental definition, 56, 227
Accidental predication, 79
——— qualities, 56, 227
Adjectives, categorematic, 28
———, attributive, 31
———, concrete and general, 31
———, abstract and general, 31
Æquipollence, 125, 129
All, ambiguous use of, 233
Ambiguity of language, 234
Ambiguous middle, 227, 232, 234
——— terms, meaning of, 41
——— terms, fallacy of, 156, 231
Ampliative judgments, 79
Analysis, 14
Analytical judgments, 67, 79, 104
——— method, 14, 217
——— train of syllogistic reasoning, 217, 219, 221
Antecedent of hypothetical propositions, 67, 74
A priori concepts and judgments, 3
Argument, 6, 120
——— in a circle, 228, 235, 237
Argumentum ad hominem, 228, 240, 241
——— ad judicium, 241
——— ad populum, 228, 240, 241
——— ad rem, 240, 241
Argumentum ad verecundiam, 228, 240, 241
——— a fortiori, 23, 120
Aristotelian sorites, 218, 221, 223, 224
Aristotle, 22, 72, 84, 180-183, 210
Assertory propositions, 67, 72, 192
Attribute, 30
Attributive, 31, 93
Attributive theory of predication, Mill's, 101
Axiom of Consistency, 22
Axiom or Principle of Sufficient Reason, 22
Axioms of syllogism, the fundamental, 23, 180, 275-285

Bailey, 254, 278
Bain, 69, 104, 107, 254, 290, 296, 297
Baynes, 14
Begging the question, 237
Boole, 69

Canons of the syllogism, 23, 180, 275-285
Categorematic words, 28
Categorical propositions, 67
——— syllogisms, 152, 193
Category or predicament, 84
Chance defined, 264
Change of relation, 125, 141-144
Class, 54, 80, 180

Collective terms, 27, 29, 41, 59, 73
———, singular and general, 29
Complex propositions, 68
Composition, fallacy of, 227, 232, 233
Compound propositions, 66, 68
Comprehension of a term, 15, 55, 93
Comte, 310
Conception, meaning of, 25
———, individual and general, 25
Concepts, *a priori*, 3
———, how formed, 26
———, objectively regarded, 4, 25
———, subjectively regarded, 2, 25
Conclusion, what, 118
———, when in a syllogism negative, 160, 175
———, when in a syllogism particular, 163, 176, 177
Concrete terms, 27, 30, 35, 41
Conditional propositions, 67–68
Conjunctive propositions, 193, 202
Conjunctive-disjunctive syllogism, 193, 202–203
Connotation of terms, 15, 36–37, 46–50
Connotation and Denotation of a term, relation between, 47
Connotative terms, 37–39, 42
Consequent of hypothetical propositions, 67, 74
Consistency, axiom of, 22
Content of a concept, 55
Contradiction, the principle or axiom of, 17, 18, 22
Contradictory terms, 52
Contraposition, 125, 132–134, 312
Contrary terms, 53
Conversion, 125–128
———, simple, 128
———, per accidens or by limitation, 128
———, by negation, or contraposition, 134
Co-ordinate species, 52
Copula, 63, 64, 65
Corr[redacted]27, 36, 41
Cross-division, 59, 227

Deductive Reasoning, 118, 119, 120, 260
Definition, what, 54, 55
———, accidental, 56, 227
———, ambiguous, 57, 227
———, circle in, 56, 227
———, complete, 55
———, figurative, 57, 227
———, incomplete or partial, 56
———, narrow, 56, 227
———, negative, 57, 227
———, obscure, 57, 227
———, redundant, 56, 227
———, rules of, 55–57
De Morgan, 254, 258, 259, 305
Denotation of terms, 37, 46–50
Denotation and connotation of a term, relation between, 47
Dependent events, 266
——— rules of, 267
Diagrams, five fundamental, 116
Dichotomy, division by, 60–61
Dictum de omni et nullo, 22, 120, 180, 275
——— de diverso, 275
——— de exemplo, 276
——— de reciproco, 276
Differentia, 52, 55, 80, 81, 82, 84
Dilemma, 202–208
———, different views of, 286
———, method of testing, 206–207
———, Ueberweg's view of, 203
Disjunctive propositions, 68–70, 142–144, 201
Disjunctive categorical syllogism, 193, 200–202
Distribution of terms, 112, 113, 114, 115, 116, 125, 157–158, 230
Division, cross-, 59, 227
———, fallacy of, 227, 232, 233
———, by dichotomy, 60–61
———, incomplete, 59, 227
———, logical, 15, 54, 55, 58–61
———, metaphysical, 58, 227

Division, overcomplete, 59, 227
———, overlapping, 59, 60, 227
———, physical, 58, 227
———, rules of, 59
Divisionis, fundamentum, 59

Enthymeme, 210
Epicheirema, 219–221
Episyllogism, 217
Episyllogistic train of reasoning, 217, 218, 221
Essential propositions, 79
Events, rules of *dependent*, 267
———, rules of *exclusive*, 266
Exceptive particles, 100
Excluded middle, law of, 17–20
Exclusive or incompatible events, 266
———, rules of, 266
Exclusive particles, 100
Experimental rules of mediate inference in probability, 266, 270
Explicative propositions, 79
Extension of a term, 15, 46, 93
Extent of a concept, 55
Extremes of a proposition, 153

Fallacia a dicto secundum quid ad dictum simpliciter, 234
Fallacia a dicto simpliciter ad dictum secundum quid, 234
Fallacy, inferential, 226, 229
———, in hypothetical categorical syllogisms, 195–196
———, logical, 226–227, 229
———, material or non-logical, 228, 234
———, meanings of, 226, 227, 228
——— of accident, 227, 233
——— of ambiguous language, 156, 227, 234
——— of ambiguous middle, 227, 232
——— of begging the question, 237
——— of composition, 227, 232
——— of division, 227, 232
——— of false premiss, 228, 237
Fallacy of four terms, 156, 234
——— of illicit process, 158, 226, 230
——— of irrelevancy or ignoratio elenchi, 228, 239–241
——— of many questions, 241
——— of non causa pro causa, 228, 237, 238
——— of non sequitur, 241
——— of petitio principii, 228, 235–238
——— of undistributed middle, 157, 226, 230
———, semilogical, 227, 232
Figure, Galen's or the fourth, 182
———, imperfect, 182
———, perfect, 181
———, special rules and valid moods of the 1st, 171, 181, 182
———, special rules and valid moods of the 2nd, 174, 175, 182
———, do. do. 3rd, 176, 182
———, do. do. 4th, 177, 182
Figures of the syllogism, 165
First figure, 166, 168–172, 181, 203, 222
Five predicables, 80
Form of thought, 8
Formal logic, 13, 15
——— truth, 13
Formally valid, 9
Fourth figure, 166, 177, 182
Fowler, Prof., 31, 65, 69, 72, 84, 287
Fundamental principles of Deductive Logic, 16–23
——— axioms of syllogism, 23, 180, 275–285
Fundamentum divisionis, 59

Galen's or the fourth figure, 182
Genera, subaltern, 85
General conception or notion, 25, 34
——— terms, 27, 28, 31, 32, 34, 37, 38, 41
Generic property, 83
Genus, 51, 80, 81, 84, 85
———, summum, 85

Hamilton, Sir W., 8, 13, 20, 22, 33, 65, 69, 72, 95, 98, 100, 105, 107, 109, 110, 280-281, 288, 289-290, 293, 303, 311
Herschel, Sir John, 254
Hobbes, 97, 109
Hypothetical propositions, 67, 68, 74, 126, 127, 128, 130, 134, 141, 143-144, 193-194
Hypothetical-categorical syllogisms, 193, 195-200
Hypothetically necessary character of all deductive reasoning, 151, 260

Identity, principle or axiom of, 16, 20, 21
Ignoratio Elenchi (or the fallacy of Irrelevancy), 228, 239-241
Illicit process, fallacy of, 158, 226, 230, 231
Immediate reasoning or inference, 118-120, 124, 265
———, kinds and forms of, 124, 125, 146-148
Imperfect figures of the syllogism, 182, 183
Import of propositions, 93
Indefinite or indesignate propositions, 73
Indirect reduction of the syllogism, 182, 183, 187-190
Individual, meaning of, 81
Individual conception, 25, 34, 55, 96
Inductive reasoning or inference, 118, 119, 120, 121, 123, 297-302
Inference or reasoning, deductive, 118, 119, 120, 260
———, defined, 118
———, immediate, 118, 119, 120, 124, 265
———, inductive, 118, 119, 120, 121, 123, 297-302
———, mediate, 118, 119, 120, 151, 266
Inference, various kinds of, 118-120, 124, 192, 193, 216-221
Inference, different meanings of, 120
Infima species, 85
Inseparable accidens, 83
Intuitive concepts and judgments, 3
Irrelevancy or Irrelevant conclusion, fallacy of, 228, 235, 239

Jevons, Prof., 31, 40, 69, 287, 306
Judgment, 2, 5, 6, 14, 63, 95, 96

Kant, 287, 302
Keynes, 4, 32, 33, 293, 314

Lambert, 23, 275-278
Language as related to thought, 6, 7
Language, logic as conversant about, 6-7
Laws of thought, 22
Lewes, 308-311
Limitation, conversion by, 128
Linguistic view of logic, 6-7
Logic, formal or pure (or logic of consistency), 13, 303
———, Hamilton's view of, 8, 303, 311
———, Lewes's view of, 307-311
———, linguistic view of, 6-7
———, material, 13, 15
———, Mill's view of, 9, 303-304
———, objective view of, 4-6, 303-312
———, postulate of, 20
———, Carveth Read's view of, 307
———, relation of, to other sciences, 11-12
———, Spencer's view of, 10, 304-308, 311, 312
———, subjective view of, 1-3
———, Ueberweg's view of, 12, 308
———, view of, adopted in this work, 10
Logic, Whately's view of, 8
Logical division, 58, 59, 60, 61
Logical machine, 306

Machine, the logical, 306

Major term, 153, 165
——, illicit process of, 158, 230, 231
Major premiss, 153
Mansel, 65, 69, 72, 96, 107, 110, 254, 286, 287
Many questions, fallacy of, 241
Many-worded terms, 27-28
Martineau, Dr James, 23, 31, 92-93, 107, 254, 255-258, 259, 278, 282
Material logic, 13, 15
—— or non-logical fallacies, 228, 234-242
Mathematical reasoning, 119, 120, 122, 226
Mediate inference, 118, 119, 120, 151, 266
Metaphysical division, 58, 227
Method, the fourth part of logic, 14
Middle, ambiguous, 227, 232, 234
—— term, 153
—— undistributed, 157, 226, 230
Mill, J. S., 9-10, 13, 20-21, 23, 31, 32, 33, 39, 40, 65, 69, 70, 97, 99, 101-106, 107, 110, 142-144, 201, 251-255, 278, 283-285, 297, 303-304, 305, 308
Minor term, 153, 165
——, illicit process of, 158, 230
Minor premiss, 153
Mixed syllogisms, 152, 192, 193, 195-207, 289-296
Mnemonic verses for the valid moods, 182
Modal consequence, 125, 140-141
Modality of propositions, 67, 71-72
Monck, 33
Moods of syllogisms, 167, 192
——, determination of valid, 168-177
——, possible, 167
——, subaltern, 171, 174
——, valid, 171, 174, 176, 177, 182

Name or term, 6, 15, 24, 25, 26, 27
Necessary propositions, 67, 71, 141, 192
Negation, conversion by, 134
Negative definition, 57, 227
—— premisses, 158, 160
—— propositions, 67, 70, 112, 114, 116
—— terms, 27, 36, 42
Non causa pro causa, fallacy of, 228, 238
Non connotative terms, 27, 39
Non-contradiction, law of, 22
Non-logical or material fallacies, 228, 234-242
Non sequitur, 241
Notion or general conception, 25
Noumenon, 25

Objective view of logic, 1-6, 302-312
Obversion, permutation or aequipollence, 125, 129, 312
Opposition of propositions, 77 78, 125, 136-140

Particular premisses, fallacy of, 163
Particular propositions, 73, 75, 113-115
Partition, physical, 58, 59, 227
Per accidens, conversion, 128
Perfect figure of the syllogism, 181
Permutation, obversion or aequipollence, 125, 129
Petitio Principii, 228, 235-237, 240, 242
Phenomenon, meaning of, 35
Physical division or partition, 58, 59, 227
Polylemma, 203, 204
Port Royal logic, 14
Positive terms, 27, 36, 42
Post hoc ergo propter hoc, 237
Postulate of logic, 20
Predicables, 80, 84
Predicament or category, 84
Predicate of a proposition, 63, 65, 80, 84, 193

Predicate, distribution of the, 113, 115, 116
———, quantification of the, 98–101
Predication, theory of, 93
Premiss, 118, 151
Premiss, major, 153
———, minor, 153
Principle of division, 59
Principle or axiom of sufficient reason, 22
Principles of deductive logic, fundamental, 16–23
——— of syllogism, fundamental, 23, 180, 275–285
Privative terms, 27, 36
Probability, 262–273
——— defined, 264
Probable proposition, the meaning of, 262–264
——— reasoning, 262
Probable syllogisms, 192
Problematic propositions, 67, 71, 72, 141
Progressive or synthetic or episyllogistic train of syllogistic reasoning, 217, 221
Proper names, 39–40, 47
Property or proprium defined, 83
Property, generic, 83
——— individual, 83
——— specific, 83
Propositions, 6, 15, 63
———, affirmative, 67, 70
———, ampliative and analytical, 67, 79, 104
———, categorical, 67, 141
———, complex, 68
———, compound, 66, 68
———, conditional, 67–68
———, conjunctive (copulative and remotive), 193, 202
———, contradictory, 78, 139
———, contrary, 78, 139
———, copula of, 63, 64, 65
———, definition of, 63
———, disjunctive, 68–70, 142–144, 201
Propositions, essential or explicative, 79
———, hypothetical, 68, 71, 74, 126, 127, 128, 130, 134, 141, 143–144, 193–194
———, import of, 93
———, indefinite or indesignate, 73
———, modality of, 67, 71–72
———, negative, 67, 70, 112, 114, 116
———, necessary, 67, 71, 72, 141, 192
———, opposition of, 77–78, 136–140
———, particular, 73, 75, 113–115
———, predicate of, 63, 65, 80, 84, 193
———, problematic, 67, 71, 72, 141
———, quality of, 67, 70, 75, 76, 194
———, quantity of, 67, 73–74, 75, 76, 193
———, real, 67, 79–80, 104
———, relation of, 67
———, single, 66
———, subalternation of, 78, 135–136
———, subcontrary, 136, 139
———, subject of, 63, 80, 193
———, synthetical, 67, 79, 104
———, universal, 67, 73, 75, 76, 110–113, 116
———, various divisions of, 67
———, verbal, 67, 79–80, 104
Proprium [see Property]
Prosyllogism, 217
Prosyllogistic (analytic or regressive) train of reasoning, 217, 219, 221
Psychology, 11, 12, 94, 121, 255
Pure logic, 13, 303
Pure syllogisms, 152, 192, 193–195

Quality of propositions, 67, 70, 75, 76, 194

Quantification of the predicate, 98–101
Quantity of propositions, 67, 73–74, 75, 76, 193

Read, Carveth, 110, 307
Real propositions, 67, 79–80, 104
Real truth, 12
Reason, axiom of sufficient, 22
Reasoning, 3, 5, 6, 15, 118–123 [see Inference]
Reasoning, probable, 262
Reductio per deductionem ad impossibile, 183, 187–190
Reduction of syllogisms, 182
——— direct or ostensive, 183–187
——— indirect, 183, 187–190
——— of hypothetical to categorical syllogisms, 194
——— of hypothetical-categorical to categorical syllogisms, 197
Regressive or prosyllogistic or analytic train of syllogistic reasoning, 217, 219, 221
Relation of propositions, 67
Relation, change of, 125, 141–144
Robertson, Prof., 14, 293
Rules of conversion, 125
——— of definition, 55–57
——— of division, 59
——— of hypothetical-categorical syllogisms, 195–196
——— of immediate inference in probability, 265
——— of inference, by opposition, 139
——— of mediate inference in probability, 266–271
——— of syllogism, general, 156–164
——— ———, special, 171, 175, 176, 177, 181

Second figure, 166, 172–175, 182, 203, 223
Semilogical fallacies, 227, 232
Separable accidens, 83
Single propositions, 66
Single-worded terms, 27–28
Singular propositions, 78
——— terms, 27, 28, 29, 32, 33, 39, 41, 59, 73
Sorites, definition of, 218
———, Aristotelian, 218, 221, 222–224
———, Goclenian, 218–219, 221, 222–224
Species, 51, 80, 81, 82
———, infima, 85
———, subaltern, 85
Spencer, Herbert, 10, 13, 110, 304–307, 310, 311, 312
Subaltern, 78
——— genera, 85
——— moods, 171, 174
——— species, 85
Subalternant, 78
Subalternate, 78
Subalternation, 78, 125, 135–136
Sub-contrary propositions, 136, 139
Subject of a proposition, 63, 80, 193
Subjective view of logic, 1–3
Subordinate terms, 52
Sufficient reason, axiom of, 22
Summum genus, 85
Superordinate terms, 52
Syllogism, characteristics of, 151–152
———, definition of, 151
———, functions and value of, 251–261
——— the charge of a petitio principii against, 253, 256–260
Syllogisms, categorical, 152, 193
———, conjunctive-disjunctive, 193, 202–206
———, disjunctive-categorical, 193, 200–202
———, figures of, 165
———, hypothetical, 193–195
———, hypothetical-categorical, 193, 195–200
———, mixed, 152, 192, 193, 195–207, 289–290

Syllogisms, moods of [see Moods]
——, pure, 152, 192, 193–195
Syllogistic rules [see Rules of Syllogism]
Syncategorematic words, 28
Synthesis, 14–15
Synthetic method, 14–15
Synthetical propositions, 67, 79–80, 104

Term, meaning of, 26–27
Terms, absolute, 27, 36, 42
——, abstract, 27, 30, 38, 39, 41
——, ambiguous, 41
——, collective, 27, 29, 41, 59, 73
——, concrete, 27, 30, 35, 41
——, connotation of, 15, 36–37, 46–50
——, connotative, 37–39, 42
——, contradictory, 52
——, contrary, 53
——, correlative, 27, 36, 41
——, denotation of, 37, 46–50
——, distribution of, 112, 113, 114, 115, 116, 125, 157–158, 230
——, general, 27, 28, 31, 32, 34, 37, 38, 41
——, many-worded, 27–28
——, negative, 27, 36, 42
——, non-connotative, 27, 39
——, positive, 27, 36, 42
——, privative, 27, 36
——, single-worded, 27–28
——, singular, 27, 28, 29, 32, 33, 39, 41, 59, 73
——, various divisions of, 27

Tet[illegible]
Theory of predication, 93
Third figure, 166, 175–176, 182, 224
Thomson, 23, 69, 278, 279, 287
Thought, different meanings of, 2
——, laws of, 22
——, its relation to language, 6, 7
Trains of reasoning, 216–224
Trilemma, 203, 204
Truth, formal, 13
——, real, 12

Ueberweg, 12, 22, 34, 69, 70, 96, 107, 110, 142–143, 201, 203, 308
Undistributed middle, fallacy of, 157, 226, 230
Universal propositions, 67, 73, 75, 76, 111–113, 116

Valid, meaning of the word, 13–14
Venn, Dr, 72, 107, 264, 265, 307–308
Verbal propositions, 67, 79, 104
Verbs, categorematic, 28

Wallis, 287
Whately, 7–8, 23, 69, 237, 238, 239–241, 280, 286, 287
Whewell, Dr, 254
Wolf, 287
Words, categorematic, 28
——, syncategorematic, 28

CAMBRIDGE: PRINTED BY J. & C. F. CLAY, AT THE UNIVERSITY PRESS.